Math 7770 Spring 2011
Homework 9
Due: April 4, 2011

You are welcome to work together but everyone needs to write up **distinct** solutions. If you use any books outside of our textbook or other people, please make sure to give them credit. Make sure your solutions are complete. If your handwriting is atrocious, I am happy to give you a basic introduction to LaTeX.

Warmup

- §9.1 #1. If $f(x) \mid g(x)$ and $g(x) \mid f(x)$, prove that there is a rational number c such that $g(x) = cf(x)$.
- §9.1 #4. If $p(x)$ is irreducible, prove that $cp(x)$ is irreducible for any rational $c \neq 0$.
- §9.4 #3. Let $\alpha = \alpha_1 + \alpha_2 i$ be an algebraic number, where α_1 and α_2 are real. Does it follow that α_1 and α_2 are algebraic numbers?

Problems

1. §9.1 #3. If $p(x)$ is irreducible and $g(x) \mid p(x)$, prove that either $g(x)$ is a constant or $g(x) = cp(x)$ for some rational number c.
2. §9.1 #6. If $f(x)$ and $g(x)$ are primitive polynomials, and if $f(x) \mid g(x)$ and $g(x) \mid f(x)$, prove that $f(x) = \pm g(x)$.
3. §9.2 #1. Find the minimal polynomial of each of the following algebraic numbers:

$$7, \sqrt[3]{7}, \frac{1+\sqrt[3]{7}}{2}, 1+\sqrt{2}+\sqrt{3}.$$

   Which of these are algebraic integers?
4. §9.4 #2. For any algebraic number α, define m as the smallest positive rational integer such that $m\alpha$ is an algebraic integer. Prove that if $b\alpha$ is an algebraic integer, where b is a rational integer, then $m \mid b$.