This is just a guide to help you study. I do not guarantee that anything will or will not be on the exam based on this guide (except the section titles "Not on the Final").

1   Basics

Monday, December 13, 2010 from 8-10:30 AM in Mendel 115 for the 8:30 section and 1:30-4 PM in Mendel G90 for the 12:30 section. No books or notes or cell phones. You may use a scientific calculator and the Unit Circle Chart. You will be given the same equations for area and volume that were on the 3rd exam.

Office Hours Friday, December 10 from 10:00AM-2:00PM in my office (318 SAC). Sunday, December 12 from 1:00-4:00 PM in 310 SAC (seminar room just down the hall from my office)

2   Suggestions

- Work lots and lots of problems, especially those on material you don't understand as well.
- When possible, ask yourself WHY you are solving a problem a certain way or WHY the result is true.
- Do not look at solutions unless you are desperate.
- Check your work!!

3   Material

3.1   Functions

We dealt with polynomials, rational, trig, logarithmic, exponential, and absolute value functions, including:
- Domain and Range
- Intercepts
- Graphs of the Basic Functions
- Symmetry: Even and Odd
- How to find inverses and what inverses mean
- Continuity
- Asymptotes
- Interpreting word problems as functions/ modeling
- The basics about trig functions and identities we use a lot (like $\sin^2 x + \cos^2 x = 1$)

3.2   Limits

Limits tell us about the behavior of a function as we get very close to a particular point (or infinity).
- If you can use Direct Substitution for finite limits, do it!
- If there is some algebra you can do to simplify the function (such as canceling a term in the numerator and denominator), do that!
- If your function is a rational function, use the rules about degrees of the numerator and denominator.
- If your limit satisfies one of the types where we can apply L'hospital's Rule, use it!
- Use the facts that $\lim\limits_{x \to \infty} \frac{1}{x} = 0$ and $\lim\limits_{x \to 0} \frac{1}{x} = \infty$ to analyze similar functions.

3.3 Derivative Rules

- Using the limit definition to find a derivative
- Power Rule (for polynomials)
- Product/Quotient Rule
- Trig Functions
- Logs and Exponential Functions
- Chain Rule
- Implicit Differentiation
- Determining information about a function from the graph of its derivative.

3.4 Tools

- Intermediate Value Theorem
- Mean Value Theorem
- Squeeze Theorem

3.5 Applications

- How to find an equation for the tangent line
- Analyzing information about the derivative of a function in a word problem (§3.7)
- Position, velocity, acceleration
- Exponential Growth and Decay
- Related Rates
- Optimization Problems
- Marginal Cost, Profit, Revenue
- Increasing-Decreasing Test and Concavity Test
- First and Second Derivative Test

3.6 Not on the Final

- How to shift graphs by adding or multiplying the function by a number
- Inverse Trig Functions and Their Derivatives
- Other bases besides e for log functions
- Hyperbolic Functions
- Logarithmic Differentiation (although you are welcome to use this if you want to)
- Using the Limit Laws to find a limit (although you will need to know how to find limits as in the section on limits above)
- Newton's Method and Linear Approximation Problems EXCEPT for Concept Question.

4 Concept Questions

One of the following four questions will be on the final (exactly as stated). You will need to carefully explain your answer in **several complete** sentences. You are welcome to use pictures and symbols to help answer the question. **One line answers will get no credit**.

1. Explain **in your own words** how we use the secant lines of a function to define the derivative at a point. Then explain why we have to go to all that trouble to determine the slope.
2. Explain **in your own words** how linear approximation works and why we can use it to estimate values of a function near a point.
3. State the Second Derivative Test and explain **in your own words** why it is true. (You may use the Increasing/Decreasing Test, the Concavity Test, and the First Derivative Test without having to state what they say.)
4. State the Intermediate Value Theorem and explain **in your own words** how and why we use it to show that a function has a root on a certain interval.

5 Practice Problems

- See the previous exam reviews for practice problems from the first three chapters.
- pg. 347 **Concept Check** 1, 4-7, 8, 9
- pg. 348-350 **Exercises** 1-18, 19-34 (identify the 7 components for the graph, you do not need to graph), 45, 49, 50, 58, 59, 60b

6 Sample Exam Questions from Chapter 4

1. Verify that the function $f(x) = 4 + \sqrt{x-1}$ on the interval $[1,5]$ satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

2. The graph of the **derivative** f' of a function f is shown below.
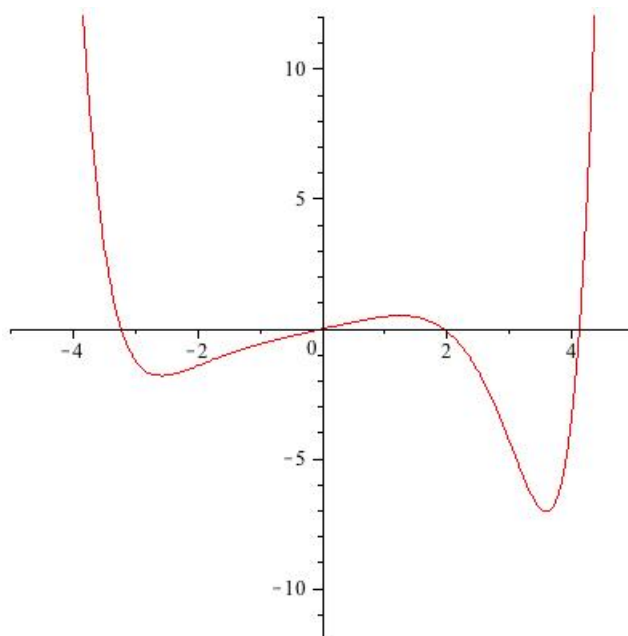
   (a) On what intervals is f increasing or decreasing?
   (b) At what values of x does f have a local maximum or minimum?
   (c) On what intervals is f concave upward or concave downward?

3. Given the function $f(x) = \frac{x}{\sqrt{x^2-1}}$

   (a) What is the domain of $f(x)$?
   (b) What are the intercepts of $f(x)$?
   (c) Is $f(x)$ even or odd or neither?

4. Given the function $f(x) = \frac{\ln x}{(x-1)^2}$.

(a) What is the domain of $f(x)$?

(b) Does $f(x)$ have any horizontal asymptotes?

(c) Does $f(x)$ have a vertical asymptote at $x = 1$?

5. Find the limits.

(a) $\lim\limits_{x \to 0} \dfrac{e^x + e^{-x} - 2}{1 - \cos 2x}$

(b) $\lim\limits_{x \to \infty} x\left(e^{\frac{1}{x}} - 1\right)$

(c) $\lim\limits_{x \to 0} (1 + 3x)^{\frac{1}{2x}}$

6. Let $f(x) = \cos x + \dfrac{\sqrt{3}}{2} x$ on the interval $0 \le x \le 2\pi$.

(a) Find the critical number(s) of the function.

(b) Find the intervals on which f is increasing or decreasing.

(c) Using both the first and second derivative test, find the local maximum and minimum values of f.

(d) Find the interval(s) where the function is concave upward and concave downward.

(e) Find the inflection point(s).

7. A cylindrical container, open at the top and of capacity 24π cubic inches is to be manufactured. If the cost of the material used for the bottom of the container is 6 cents per square inch, and the cost of the material used for the curved part is 2 cents per square inch, find the dimension which will minimize the cost. (Hint: The bottom of the cylinder is a circle with area πr^2 and the curved part is really a rectangle–visualize cutting open a can and unfolding the curved part – with height h and length the circumference of the bottom which is $2\pi r$.)