Basic Information

This assignment is due in Gradescope by 10 PM on the dates below.

Make sure you understand MHC honor code and have carefully read and understood the additional information on the class syllabus and the grading rubric. I am happy to discuss any questions or concerns you have!

You are always welcome to ask me for small hints or suggestions on problems.

Problems

Reading Problem 5M (Due: Sunday, March 1)

In class a bit ago, we showed that for each $n \in \mathbb{Z}$ we have n^2 is either a multiple of 3 or a multiple of 3 plus 1. Restate this statement in terms of congruences instead.

Wednesday Problems HW5 (Due: Wednesday, March 4)

Be sure to use the techniques and proof-writing guidelines we have talked about in class.

1. Let a, b, c, d, f be positive integers. If gcd(a, b) = d and gcd(a, c) = f and gcd(b, c) = 1, prove that gcd (d, f) = 1.

2. In class, we defined a function $f : A \to B$ to be surjective if

$$\textit{for all } b \in B \textit{ there exists an } a \in A \textit{ so that } f(a) = b$$

(a) Carefully write the negation of this statement.
(b) Use (a) to prove that the function $g : \mathbb{Z} \to \mathbb{Z}$ satisfying $g(n) = 2n + 1$ is not surjective.

3. We use the symbol $\mathbb{R}^+$ to mean all positive real numbers.
(a) Prove that the function $f : \mathbb{R}^+ \to \mathbb{R}^+$ defined as $f(x) = \sqrt{x}$ is injective.
(b) Prove that the same function in (a) is surjective.

4. Above you showed that $g : \mathbb{Z} \to \mathbb{Z}$ satisfying $g(n) = 2n + 1$ is not surjective. Instead prove that $h : \mathbb{R} \to \mathbb{R}$ satisfying $h(x) = 2x + 1$ is surjective.

5. Let $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ be the piecewise function defined as $f(x) = \begin{cases} 1 & \text{if } x = 1 \\ x - 1 & \text{if } x > 1 \end{cases}$ (remember that $\mathbb{Z}^+$ means the positive integers).

(a) Is this function injective? Prove or disprove your claim.
(b) Is this function surjective? Prove or disprove your claim.

6. For this problem , let A and B both be *finite* sets. Explain your answers in an informal way (as if you were explaining to a friend who doesn't take any math classes). I am not looking for a formal proof, but write your answers in several complete sentences.
   (a) If $f : A \to B$ is *injective*, what can we say about $|A|$ versus $|B|$? Why?
   (b) If $f : A \to B$ is *surjective*, what can we say about $|A|$ versus $|B|$? Why?
   (c) If $f : A \to B$ is *bijective*, what can we say about $|A|$ versus $|B|$? Why?

Reading Problem 5F (Due: Thursday, March 5)

Suppose we want to prove that for all $n \in \mathbb{Z}^+$ we have $1 + 3 + 5 + \cdots + (2n-1) = n^2$ using induction. What do you think the base case is? What is the induction hypothesis? (You are not being asked to prove the result.)