MATH 232 Discrete Math

Homework 4

Basic Information

This assignment is due in Gradescope by 10 PM on the dates below.

Make sure you understand MHC honor code and have carefully read and understood the additional information on the class syllabus and the grading rubric. I am happy to discuss any questions or concerns you have!

You are always welcome to ask me for small hints or suggestions on problems.

Problems

Reading Problem 4M (Due: Sunday, February 22)

(This was posted last week.) Find an example of functions f and g so that $g \circ f$ is surjective, and g is surjective, but f is not surjective. Your example can be ANY functions f and g. (This is one of the example mentioned on page 333 of MR.)

Wednesday Problems HW4 (Due: Wednesday, February 25)

Be sure to use the techniques and proof-writing guidelines we have talked about in class.

1. Let $A = \{3x + 1 : x \in \mathbb{R}\}$ and $B = \{3x - 2 : x \in \mathbb{R}\}$. Prove that $A = B$.

2. Given two sets A and B, we say they are *disjoint* if they have no elements in common. We want to prove that, given any sets A and B, the sets $A \cap B$ and $A - B$ are disjoint.
   (a) How would we write down "for all", "there exist", and/or "if…then…" statements to precisely capture what it would mean for these two sets to be disjoint?
   (b) Now prove the statement(s) you wrote down in (a).

3. (a) Prove that if $ab \mid c$ then $a \mid c$ and $b \mid c$.
   (b) For all $m, n \in \mathbb{Z}$ prove that the set $A = \{x \in \mathbb{Z} : m \cdot n \mid x\}$ is a subset of the set $B = \{x \in \mathbb{Z} : m \mid x\} \cap \{x \in \mathbb{Z} : n \mid x\}$.

4. Let a, b be positive integers. Prove that if $\gcd(a, b) > 1$ then $b \mid a$ or b is not prime. (Hint: how do we prove *or* statements?)

5. The following claim is false:

Let A, B, and C be subsets of some set U. If $A \nsubseteq B$ and $B \nsubseteq C$, then $A \nsubseteq C$.

(a) Here is a wrong "proof". Describe precisely where the logic fails in this proof.

We assume that A, B, and C are subsets of U and that $A \nsubseteq B$ and $B \nsubseteq C$. This means that there exists an element $x \in A$ that is not in B and there exists an element x that is in B and not in C. Therefore, $x \in A$ and $x \notin C$, and we have proved that $A \nsubseteq C$.

(b) Suppose we are in a special case of (a) where the set U is the integers $\mathbb{Z}$. Come up with an explicit example for sets A, B, and C where the claim stated above fails. If you work with others to come up with ideas for this part, you should each have different final answers. Briefly say why your example is correct.

6. Prove that if n is an odd integer, then $n^2 = 8x + 1$ for some $x \in \mathbb{Z}$.

7. For this problem, you can simply write down each algebraic step you do. It is ok to NOT use any words in the solution to this problem! But make sure you show every step of the computation. (This is problem 5d in MR, pg 424.)

(a) For the integers $a = 21361$ and $b = 12628$, use the Euclidean algorithm to find $\gcd(a, b)$.

(b) Use the algorithm "backwards" to write the $\gcd(a, b)$ as a linear combination of a and b.

Reading Problem 4F (Due: Thursday, February 26)

In class we showed that for each $n \in \mathbb{Z}$ we have n^2 is either a multiple of 3 or a multiple of 3 plus 1. Restate this statement in terms of congruences instead.