

Basic Information

This assignment is due in Gradescope by 10 PM on the dates below.

Make sure you understand MHC [honor code](#) and have carefully read and understood the additional information on the [class syllabus](#) and the [grading rubric](#). I am happy to discuss any questions or concerns you have!

You are always welcome to ask me for small hints or suggestions on problems.

Problems

Reading Problem 10M (Due: Sunday, April 26)

What happens to leaves in Prüfer's method of assigning a sequence to a labeled tree?

Wednesday Problems HW10 (Due: Wednesday, April 29)

Be sure you completely justify your answer using properties or results from class. An answer without justification will earn 0 points.

1. Toss a dice 5 times.

- (a) What is the probability that you will get the same number on all the rolls?
- (b) What is the probability that each number will be different?

2. What is the probability of finding two people who were born in the same month of the year in a group of six randomly selected people?

3. A box contains 6 balls: 2 red, 3 green, and 1 black. You remove two balls, one after the other.

- (a) What is the probability that no green balls were drawn?
- (b) What is the probability that if the first ball is red, no green balls were drawn?

4. Suppose that a laboratory test will be positive for 90% of the subjects who have a disease and negative for 95% of those who do not have the disease. Suppose also that 3% of the population has the disease.
- (a) If a randomly selected person is tested, and the result is positive, what is the probability that the person actually has the disease?
 - (b) If the result is negative, what is the probability that the person does not have the disease?
5. Prove that for any graph G of order at least 2, the degree sequence has at least one pair of repeated entries.
6. Let n be a positive integer and G be a bipartite graph with partites X and Y . If the degree of all vertices in X are all $\geq n$ and degree of all vertices in Y are all $\leq n$ then prove the number of vertices in X at is at most equal to the number of vertices in Y .
7. Let T be a tree such that every vertex adjacent to a leaf has degree at least 3. Prove that some pair of leaves in T has a common neighbor (in other words, both leaves are adjacent to the same vertex). **Hint:** One way to prove this is to consider the longest path through the tree.

Reading Problem 10F (Due: Friday, May 1)

Given a graph G with at least one bridge, what can we say about whether the graph has a Hamiltonian cycle?