MATH 311 Advanced Linear Algebra
Homework 8

Basic Information
This assignment is due in the correct folder in Google Drive by **4 PM on Friday, April 11**. Any part of the assignment you LaTeX can be turned in by 10 PM without penalty.

Make sure you understand MHC honor code and have carefully read and understood the additional information on the class syllabus and the grading rubric. I am happy to discuss any questions or concerns you have!

You are always welcome to ask me for small hints or suggestions on problems.

Problems
1. P.8.18

2. (a) P.9.7 (do part (b) before you answer the "Discuss" part)
   (b) Do the same problem as in (a) but for the matrices $A = \begin{bmatrix} -1 & 4 \\ 2 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 6 \\ 3 & 1 \end{bmatrix}$. Why is this example different? ("Discuss" the differences between (a) and (b).)

3. (a) P.9.21 Prove that the only eigenvalues of an idempotent matrix are $\lambda = 0$ and $\lambda = 1$.
   (b) Prove that if a matrix only has eigenvalues $\lambda = 0$ and $\lambda = 1$ then it must be idempotent.

4. P.10.11

5. This problem will be worth 6 points.
   (a) P.10.36
   (b) P.10.37

We can use Python to compute eigenvalues. I've updated the Python guide to explain one way to do that (using numpy linalg package). Use Python to help you solve the following problem. Remember, you should submit your code as .py files. See the "Where to save work and how to submit it" section of the Python guide.

6. This problem will also be worth 6 points. Make up a few 4×4 and 5×5 matrices with coefficients in $\mathbb{R}$ and have Python compute their eigenvalues. You should compute at least 3 matrices of each size, and if they don't have any complex eigenvalues, try different matrices. Based on what you observe about the eigenvalues Python computed for you, answer the following questions (with basic explanation).
   (a) If I give you matrix $A \in M_{10}(\mathbb{R})$ and I tell you five of its eigenvalues are:
   $$3 + 2i \quad 4 - i \quad -5 + 4i \quad -i \quad 1 + i$$
   without doing any computation, what are the other 5 eigenvalues? Why? What pattern did you observe in your examples that led you to this conclusion?
   (b) Why do we know that if $A \in M_9(\mathbb{R})$ it must have a real eigenvalue? Again, connect your answer to a pattern you saw in your computations above.
   (c) Based on your answer to (a) and (b), why can we conclude that if a 3×3 *real* matrix has a complex (non-real) eigenvalue, then it must have 3 distinct eigenvalues?