1. Prove by induction on $n \geq 5$ that $2^n > n^2$.

2. Prove by induction on $n \geq 0$ that for every $a \neq 1$

$$\sum_{i=1}^{n} i a^i = \frac{n a^{n+2} - (n+1)a^{n+1} + a}{(a-1)^2}.$$

3. Prove by induction on n that for all even n, if $k \neq n/2$ then

$$\binom{n}{\frac{n}{2}} > \binom{n}{k}.$$

4. What is wrong with the following reasoning:

All cats are the same color. We prove this by induction. If there is one cat, it is the same color as itself. So assume that we have n cats, labeled 1 through n and assume by the induction hypothesis that $n-1$ cats are the same color. Cats 1 through $n-1$ are therefore all the same color, suppose they are all orange cats. Then cat 2 is orange and so by the induction hypothesis so are cats 2 through n (since this is a collection of $n-1$ cats and so they must all be the same color by the induction hypothesis). Thus all cats are orange.