Math 704 Summer 2008
Homework 1
Due: June 16, 2008

You are welcome to work together but everyone needs to write up **distinct** solutions. If you use any books or other people heavily, please make sure to give them credit. Make sure your solutions are complete. If your handwriting is atrocious, I am happy to give you a basic introduction to LaTeX.

1. Section 1.1 #20. For x an element in G show that x and x^{-1} have the same order.
2. Section 1.1 #24. If a and b are commuting elements of G, prove that $(ab)^n = a^n b^n$ for all $n \in \mathbb{Z}$. [Hint: Induction on positive n first.]
3. Section 1.1 #25. Prove that if $x^2 = 1$ for all $x \in G$ then G is abelian.
4. Section 1.3 #10. Prove that if σ is the m-cycle $(a_1\ a_2\ \dots\ a_m)$, then for all $i \in \{1, 2, \dots, m\}$, $\sigma^i(a_k) = a_{k+i}$ where $k + i$ is replaced by its least positive residue mod m. Deduce that $|\sigma| = m$.
5. Section 1.3 #15. Prove that the order of an element in S_n equals the least common multiple of the lengths of cycles in its cycle decomposition. [Hint: Use 1.1 24 and 1.3 10].
6. Find the maximum possible order of an element in S_{12} and exhibit an element of that order.
7. Section 1.6 #9. Prove that D_{24} anad S_4 are not isomorphic.
8. Section 1.7 #16. Let G be any group and let $A = G$. Show that the maps defined by $g \cdot a = gag^{-1}$ for all $g \cdot a \in G$ satisfy the axioms of a (left) group action of G on itself.