Math 321: Foundations of Abstract Algebra

Homework 4 : Due February 21

1. Let G be a group and $H < G$ where $H \neq G$. Prove that the set $S = G - H$ (the complement of H relative to G) is a set of generators of G.

2. Let $\alpha \in S_n$ with $\alpha = \alpha_1 \alpha_2 \cdots \alpha_r$ where the α_i are disjoint cycles. Prove that

$$o(\alpha) = \text{lcm}(o(\alpha_1), o(\alpha_2), \ldots, o(\alpha_r)).$$

3. As always, be sure to carefully justify your results.

   (a) # 8.11

   (b) # 8.12

4. (a) Let α be the 12-cycle $(1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12)$. For which positive integers i is α^i also a 12-cycle?

   (b) If γ is an m-cycle, m a positive integer, for which positive integers i is γ^i also an m-cycle?

5. If a permutation α is odd, prove that α^{-1} is odd.

6. # 8.27

7. Prove that for $n \geq 3$, the group A_n can be generated by 3-cycles.

8. #9.3

9. # 9.12