
Math 218: Elementary Number Theory

HOMEWORK 10 : DUE OCTOBER 14

- 2.3 #4. **Not to turn in but good practice for modular arithmetic (arithmetic in Z_m).** **Without using any computer/calculator aid** calculate $(m - 1)! \pmod m$ for $m = 4, 5, 8, 9, 10, 12, 14,$ and 15 . Be clever and use the arithmetic of congruences to your advantage.
- 2.3 #8. (a) Prove that if m is composite and greater than 4, then $(m - 1)! \equiv 0 \pmod m$.
(b) Use (a) and problem 2.3 #4 above to prove the converse of Wilson's Theorem.
- 2.3 #11. If p is prime, prove that $(p - 2)! \equiv 1 \pmod p$.
- 2.4 #2. Prove that $\phi(n)$ is even if $n > 2$.
- 2.4 #8. Suppose p is prime.
(a) Prove that if $p^2 \mid n$ then $\phi(n) = p\phi(n/p)$.
(b) Prove that if $p \mid n$ but $p^2 \nmid n$ then $\phi(n) = (p - 1)\phi(n/p)$.
(c) Use (a) and (b) to prove that if n is odd then $\phi(2n) = \phi(n)$ and if n is even then $\phi(2n) = 2\phi(n)$.