Math 215: Linear Algebra

Writing Assignment 4 : Due December 10

Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on PWeb on the given day by 3 PM or by 7 PM if you LaTeX it.

This is a writing assignment. You should treat this like you would a writing assignment in an English or Philosophy or History course, in the sense that everything you write should be part of a complete sentence and part of a larger paragraph which serves a clear purpose, your grammar and spelling should be accurate, and (if you handwrite it) you should not have crossed out sections where you change your mind about what you want say. For each problem you should plan out how you want to write it in an outline or draft, and then write a polished, final product to be submitted.

This assignment has formal proofs on it. Make sure you write them carefully and completely.

1. Let $V = \mathbb{R}$ with scalar multiplication defined as multiplication of elements in $\mathbb{R}$ and addition defined as $x + y = \max\{x, y\}$. In this problem you will prove that there is no vector which could be the zero vector of this set (and hence it is not a vector space).

   (a) By part (5) of the definition of vector space, a zero vector existing would mean: *There is a vector $\vec{z} \in V$ so that for all vectors $\vec{v} \in V$ we have $\vec{v} + \vec{z} = \vec{v}$.* Negate this statement.

   (b) Prove your statement from (a) and conclude V is not a vector space.

2. Let V be a vector space. (Hint: The order of these questions probably matters.)

   (a) Let $c \in \mathbb{R}$ and $\vec{v} \in V$. Carefully prove that if $c \cdot \vec{v} = \vec{0}$ then $c = 0$ or $\vec{v} = \vec{0}$. To prove an "or" statement in mathematics, we assume that the first statement is false (since if it is true, we have proven the result) and show the second statement must be true in this case.

   (b) Let $c, d \in \mathbb{R}$ and $\vec{v} \in V$. Assume $c \cdot \vec{v} = d \cdot \vec{v}$. Carefully prove that if $\vec{v} \neq \vec{0}$ then $c = d$.

   (c) Carefully prove that if V has more than one element, then it must have an infinite number of elements. (Suggestion: Consider the set $S = \{r\vec{v} : r \in \mathbb{R}\}$.)