Math 215: Linear Algebra

Writing Assignment 2 : Due November 10

Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by the 3 PM or by 7 PM if you LaTeX it, submit it on PWeb.

This is a writing assignment. You should treat this like you would a writing assignment in an English or Philosophy or History course, in the sense that everything you write should be part of a complete sentence and part of larger paragraphs which serve a clear purpose, your grammar and spelling should be accurate, and (if you handwrite it) you should not have crossed out sections where you change your mind about what you want say. For each problem you should plan out how you want to write it in an outline or draft, and then write a polished, final product to be submitted. Your audience should be fellow students who are excited about math but have not learned linear algebra yet. You are welcome to ask a friend who is not in the class to read your answer and let you know if it makes sense to them. There is no page or paragraph limits for this assignment but you should be (1) thorough and complete and (2) concise and exact.

1. For this problem assume that A and B are two finite sets (so they have a finite number of elements in them). For both parts of this problem, you should justify your answer with careful mathematics. As the instructions at the top of the page say, plan your answer out carefully before you start writing, and make sure you remember the audience you are writing for.

   (a) Carefully describe what we can say about the size of $A \times B$, the Cartesian product.

   (b) Suppose $f : A \to B$ is a function. Carefully describe what we can say about the relative sizes of A and B if the function is injective and, then separately if the function is surjective.

   For the next problem, I ask you to write proofs. You should still follow the directions above and not have anything crossed out, but you may want to consider the audience to be a fellow student in the class, so you can make assumptions about what the student knows from Calculus, and write for all proofs in the formal way we discuss in class.

2. This problem asks you to prove a result we will use much later in the semester. Let $V = \mathbb{R}^3$. If $\vec{v}$ and $\vec{w}$ are both elements of V, then the symbol $\langle \vec{v}, \vec{w} \rangle$ represents the dot product of these two elements (see Problem Set 1 for the definition of the dot product). Prove each of the following statements.

   (1) For all $\vec{v} \in V$, we have $\langle \vec{v}, \vec{v} \rangle \geq 0$.

   (2) For all $\vec{v}, \vec{w} \in V$, we have $\langle \vec{v}, \vec{w} \rangle = \langle \vec{w}, \vec{v} \rangle$.

   (3) For all $\vec{v}, \vec{w}, \vec{u} \in \mathbb{R}^3$ and all $c \in \mathbb{R}$, we have $\langle c\vec{v} + \vec{w}, \vec{u} \rangle = c\langle \vec{v}, \vec{u} \rangle + \langle \vec{w}, \vec{u} \rangle$.