Math 215: Linear Algebra

Problem Set 5 : Due November 9

(16 points) Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by 3 PM Grinnell Time, or 7 PM if you LaTeX the assignment

1. (2 points) In class, we defined the identity function $\mathrm{id}_A : A \to A$ as $\mathrm{id}_A(a) = a$. When $A = \mathbb{Z}$, explicitly describe this function as a subset of the product $\mathbb{Z} \times \mathbb{Z}$. Your answer should be written in set notation either as "carved out" from another set (so $\{x \in S : P(x)\}$) or "parametrically" (like $\{f(x) : x \in S\}$).

2. (4 points) In class, we defined a function $f : A \to B$ to be surjective when *for all $b \in B$ there exists an $a \in A$ so that $f(a) = b$.*

   (a) Carefully write the negation of this statement.

   (b) Use (a) to prove that the function $g : \mathbb{Z} \to \mathbb{Z}$ satisfying $g(m) = 2m + 1$ is not surjective.

3. (4 points) For each of the following questions, be sure to explain your work.

   (a) Find an example of some $\vec{u} \in \mathbb{R}^2$ so that $\mathrm{Span}(\vec{u})$ is the solution set of the equation $4x - 7y = 0$.

   (b) Find an example of a, b, and $c \in \mathbb{R}$ so that $ax + by = c$ has solution set $\mathrm{Span}(\binom{3}{2})$.

4. (6 points) We will prove the following statement.

   Let $\vec{u} \in \mathbb{R}^2$. If $\vec{w} \in \mathrm{Span}(\vec{u})$ then $\mathrm{Span}(\vec{w}) \subseteq \mathrm{Span}(\vec{u})$.

   I have set up the outline of the proof. You should either fill in the blanks or you may write your own proof from scratch, but it should look very similar to my outline. **If you fill in the blanks, please underline or color differently the filled in blanks on your submitted answer.**

   We will prove this by assuming that $\vec{w} \in \mathrm{Span}(\vec{u})$ and showing ________________________. To show containment of sets, we must show that for all ________________________ in $\mathrm{Span}(\vec{w})$, $\vec{v}$ is also in ________________________.Let $\vec{v} \in \mathrm{Span}(\vec{w})$ be arbitrary. Since $\vec{w} \in \mathrm{Span}(\vec{u})$, we can ________________________. Since $\vec{v} \in \mathrm{Span}(\vec{w})$ we can ____________________. Now notice that $\vec{v} =$ ____________________. Since ________________________ $\in \mathbb{R}$ we conclude that $\vec{v} \in \mathrm{Span}(\vec{u})$. Since $\vec{v}$ was arbitrary, we have proven the containment, and so the original statement is true.