Math 215: Linear Algebra

Problem Set 17 : Due December 17

(27 points) Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by 3 PM Grinnell Time, or 7 PM if you LaTeX the assignment. Make sure you show all your calculations. **Continue to show all your row reductions.**

1. Define a linear transformation $T : P_2 \to \mathbb{R}^2$ as $T(f(x)) = \begin{pmatrix} f(3) \\ f(0) \end{pmatrix}$. Let $\alpha = (x^2, x, 1)$ be a basis of P_2, ε_2 the standard basis of $\mathbb{R}^2$, and $\beta = \left(\binom{1}{-1}, \binom{2}{1}\right)$ another basis of $\mathbb{R}^2$.
   (a) (3 points) What is $[T]_\alpha^{\varepsilon_2}$? Show your computations.
   (b) (3 points) What is $[T]_\alpha^\beta$? Show your computations.

2. Determine the column space, Col(A) for the following matrices and then determine the rank of each. Show your work and explain your answer.
   (a) (3 points) $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 6 & 3 \\ 0 & 2 & 5 \end{pmatrix}$
   (b) (3 points) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 4 & 8 \end{pmatrix}$

3. Let $T : \mathbb{R}^5 \to \mathbb{R}^4$ be a linear transformation with
$$[T]_{\varepsilon_5}^{\varepsilon_4} = \begin{pmatrix} 1 & 4 & 8 & -3 & -7 \\ -1 & 2 & 7 & 3 & 4 \\ -2 & 2 & 9 & 5 & 5 \\ 3 & 6 & 9 & -5 & -2 \end{pmatrix}.$$
   (a) (3 points) Determine the Null(T).
   (b) (3 points) Determine range(T).
   (c) (2 points) What is the rank and nullity of T?

4. Let $A = \begin{pmatrix} 1 & 4 & 3 \\ -1 & -2 & 0 \\ 2 & 2 & 3 \end{pmatrix}$.
   (a) (3 points) Compute det(A).
   (b) (4 points) Is A invertible? If so, find its inverse. If not, say why not.

5. (DO NOT TURN IN.) Explain why there is no injective linear transformation $T : P_4 \to \mathcal{M}_{2\times 2}$.

6. (DO NOT TURN IN.) Let A be an $m \times n$ matrix and let B and C be $n \times p$ matrices. Prove that $A(B + C) = AB + AC$. (Suggestion: The formula for general matrix multiplication is your friend!)

7. (DO NOT TURN IN.) Let $A = \begin{pmatrix} -1 & -3 & 0 & 1 \\ 3 & 5 & 8 & -3 \\ -2 & -6 & 3 & 2 \\ 0 & -1 & 2 & 1 \end{pmatrix}$.

   (a) Compute $\det(A)$.

   (b) Is A invertible? If so, find its inverse. If not, say why not.

8. (DO NOT TURN IN.) Let a, b, c, x, y, be arbitrary elements of $\mathbb{R}$. Define $A = \begin{pmatrix} a & b & c \\ a+x & b+x & c+x \\ a+y & b+y & c+y \end{pmatrix}$.

   Prove that $\det(A) = 0$ regardless of the values of a, b, c, x, and y.