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# Math 215: Linear Algebra

## PROBLEM SET 14 : DUE DECEMBER 4

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(26 points) Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by 3 PM Grinnell Time, or 7 PM if you LaTeX the assignment. **For this assignment, show all of your computational work.**

1. For this problem let  $V = \mathcal{M}_{2 \times 2}$  be the vector space defined in Example 4.1.4 in the textbook. For each of the following subsets  $W$  of  $V$ , either prove  $W$  is a subspace (prove it satisfies the three conditions of the definition) or explicitly explain why it is not.

(a) (3 points)  $W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{R} \text{ and } a = 1 \right\}$

(b) (3 points)  $W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{R} \text{ so that } b = 0 \text{ or } c = 0 \right\}$ . (Remember “or” in math means one or both options can happen.)

2. (4 points) Let  $V = P_3$ , defined in the discussion immediately following definition 4.1.13 in your textbook, and let  $W = \{f(x) \in V : f'(x) \in P_1\}$  where by  $f'(x)$  we mean the first derivative of the function  $f(x)$ . Either prove  $W$  is a subspace of  $V$  or explicitly explain why it is not.

3. (a) (2 points) Suppose a system of linear equations has augmented matrix

$$\begin{pmatrix} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix}.$$

Determine the solution set represented by this matrix.

- (b) (2 points) Suppose a system of linear equations has augmented matrix

$$\begin{pmatrix} -3 & 1 & 4 & 5 & 2 & 1 \\ 0 & 1 & -1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 & 1 & -2 \end{pmatrix}.$$

Determine the solution set represented by this matrix.

4. (6 points) Completely describe the set of solutions to the following system:

$$\begin{aligned}3x_2 - 6x_3 + 6x_4 + 4x_5 &= -5 \\3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 &= 9 \\3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 &= 15.\end{aligned}$$

5. (6 points) Completely describe the set of solutions to the following system:

$$\begin{aligned}x - y + 2z &= 4 \\2x + 3y - z &= 1 \\7x + 3y + 4z &= 7.\end{aligned}$$