Math 195: Demystifying Mathematics

Homework 4 : Due February 21

As with last week, for both these problems, you should write your answer in *words and sentences and paragraphs*, not as a bunch of math symbols. Imagine you are explaining to a friend why the statements below are true and use words (and maybe the occasional judicious math symbol). Think of telling a story to explain the answers in your own words. While being correct is generally important (!), I am mainly looking for good, well articulated ideas.

1. In class, we learned that every positive integer larger than 1 can be written as a product of primes. So $24 = 2^3 \cdot 3$ and $25 = 5^2$. In formal math language we say:

   any positive integer $n > 1$ is of the form $n = p_1^{a_1} \cdots p_k^{a_k}$ where the p_i are primes and the a_k are integers greater than 0.

   (a) In class we talked about how the greatest common divisor of two numbers a and b is the primes which show up in both prime factorizations raised to the minimum exponent of each prime which shows up in the two numbers (do a couple examples yourself if you don't exactly remember that result). What about the *least common multiple* (the smallest number which both a and b divide)? What general statement can you make about the primes which make up $\mathrm{lcm}(a, b)$? Try a bunch of examples before you write your answer down.

   (b) The *divisors* of an integer $n > 1$ are the positive numbers which divide n (including 1 and n). So 12's divisors are the six integers $\{1, 2, 3, 4, 6, 12\}$. Write down 10-12 numbers between 2 and 99, write all their divisors down, and record the total number of divisors. Was that total number of divisors always an even number? If so, can you find an integer n with an odd number of divisors?

   If in the end, you found an example with an odd number of divisors, explain exactly which numbers have an odd number of divisors. If you don't believe there are any integers > 1 with an odd number of divisors, explain why that cannot happen.

2. Chapter 6 in Orlin's book talks about the power of triangles and how they show up in human building in many different places. But in nature, the regular hexagons often takes a starring design role. Think honeycombs, ants' eyes, and collections of bubbles. Why the hexagon? Two reasons. (1) The regular hexagon is one of only three regular polygons which can tile the plane (be put together without any holes between the shapes–see page 50 in our book) and (2) regular hexagons are "interior heavy," of those three shapes from (1), they have the largest interior area for a fixed perimeter.

   Your goal for this problem is to explain why (2) is true. A set of steps you may want to consider to solve this problem are outlined below. Once you have completed those steps, explain as if to a friend what you did at each step and *why* that tells us that hexagons are "more efficient" shapes than squares or triangles to tile space.

- You may assume (1) is true and the only regular shapes which can tile are triangles, squares, and hexagons. (If you want to think about why this is true, it has to do with divisors of 360.)
- Draw a regular (also called equilateral) triangle, a square, and a regular hexagon. Remember *regular* means all the sides are the same exact length.
- Assume each one has perimeter 1 unit. How long does that make each side in each case?
- Now compute the area of the three objects whose side lengths you found above. Triangle and square should be relatively straightforward. For the hexagon, can you make triangles? It's fine to use a calculator to do some of the basic arithmetic with fractions for you.
- Which one has the largest area?