November 18, 2019

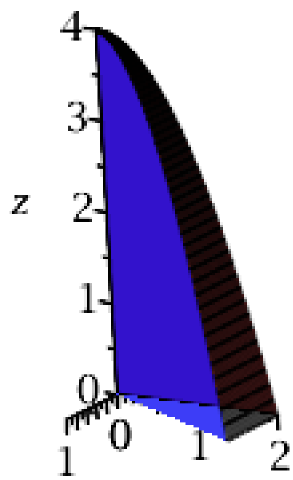
Today we are going to use Maple to visualize several surfaces and integrate over these surfaces using triple integrals. We will need the special command *display*, so hit enter on the next line. The *display* command will allow us to plot on one graph all the different functions which define the boundaries of the region.

$with(plots):$

The Example from Class

In class on Friday we started to compute $\iiint_E xyz \, dV$ where E is the region bounded by the functions $z = 4 - y^2$, $y = 2x$, $z = 0$, and $x = 0$. Here is the command I used to plot the picture from class. Hit enter on the line below.

$$display\left(\left[plot3d([0, y, z], y = 0..2, z = 0..4 - y^2, color = red, style = surface, transparency = 0.25), plot3d\left(\left[\frac{y}{2}, y, z\right], y = 0..2, z = 0..4 - y^2, color = blue, style = surface, transparency = 0.25\right), plot3d\left([x, y, 0], x = 0..\frac{y}{2}, y = 0..2, color = gray, style = surface\right), plot3d\left([x, y, 4 - y^2], x = 0..\frac{y}{2}, y = 0..2, color = black, style = wireframe\right)\right], axes = normal, labels = ['x', 'y', 'z'], scaling = constrained\right)$$

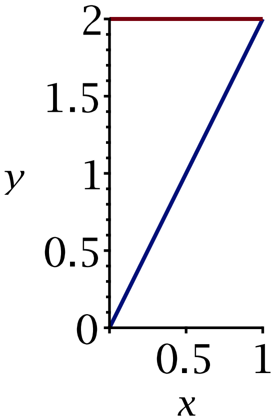


There are six possible orderings for this triple integral. We talkes about two of the integrals in class Friday. Today we set up the other four.

Project onto the xy-plane

These are the integrals we discussed in class. When we considered the projection onto the *xy*-plane we got the following graph:

$plot([2\,x, 2], x = 0..1, labels = ['x','y'], scaling = constrained)$

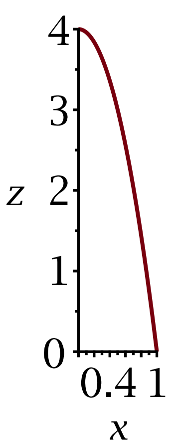


$$\int_0^2 \int_{x=0}^{x=\frac{y}{2}} \int_{z=0}^{z=4-y^2} xyz \; dz\, dx\, dy \quad \text{and} \quad \int_0^1 \int_{y=2x}^{y=2} \int_{z=0}^{z=4-y^2} xyz \; dz\, dy\, dx$$

Project onto the xz-plane

Here is the proejction onto the xz-plane:

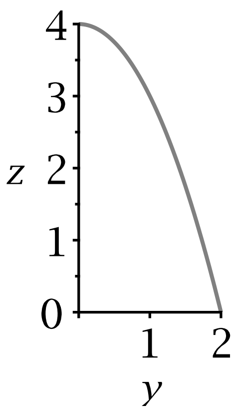
$plot([4-(2\,x)^2], x = 0..1, labels = ['x','z'], scaling = constrained)$



$$\int_{0.}^{4} \int_0^{x=\frac{\sqrt{4-z}}{2}} \int_{y=2x}^{y=\sqrt{4-z}} xyz\, dy\, dx\, dz \quad \text{and} \quad \int_0^1 \int_0^{4-(2x)^2} \int_{y=2x}^{y=\sqrt{4-z}} xyz\, dy\, dz\, dx$$

Project onto the yz-plane

$plot(4 - y^2, y = 0..2,\ labels = ['y','z'], scaling = constrained)$



$$\int_0^4 \int_0^{\sqrt{4-z}} \int_0^{\frac{y}{2}} xyz \, dx \, dy \, dz \quad \text{and} \quad \int_0^2 \int_0^{4-y^2} \int_0^{\frac{y}{2}} xyz \, dx \, dz \, dy$$

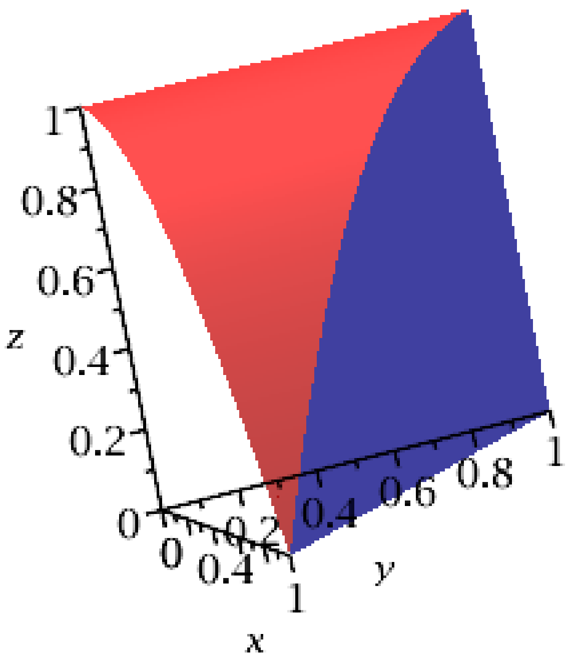
Another Example

Let's try a different example. Set up the 6 integrals for $\iiint_E xy + z \, dV$ where E is the region in the first octant bounded by $z = 1 - x^2$, and $y = 1 - x$.

$A := plot3d\left([1 - x^2], x = 0..1, y = 0..1 - x, axes = normal, color = red,\ style = surface, transparency = 0.25, labels = ['x','y','z'] \right):$

$B := plot3d([x, 1 - x, z], x = 0..1, z = 0..1 - x^2, color = blue, style = surface, transparency = 0.25):$

$display(A, B)$



Project onto the xy-plane

First fix *z* and set up the two integrals in this direction. Fixing z would show us a red right triangle with height of 1, a base length of 1.

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$$\int_{x=0}^{x=1}\int_{y=0}^{y=1-x}\int_{z=0}^{z=1-x^2} f(x, y, z)\, dzdydx \quad \text{and} \quad \int_{y=0}^{y=1}\int_{x=0}^{x=1-y,}\int_{z=0}^{z=1-x^2} f(x, y, z)\, dzdxdy$$

Project onto the xz-plane

Now fix *y* and below set up the two integrals in this direction. Fill in both integrals below.

$$\int_{z=0}^{z=1}\int_{x=0}^{x=\sqrt{1-z}}\int_{y=0}^{y=1-x} f(x, y, z)\, dydxdz \quad \text{and} \quad \int_{x=0}^{x=1}\int_{z=0}^{z=1-x^2}\int_{y=0}^{y=1-x} f(x, y, z)\, dydzdx$$

Project onto the yz-plane

This one needs to be split into two integrals: one under the red part and one under the blue part.

$$\iiint_E xy+z\,\mathrm{dV} = \int_{z=0}^{z=1}\int_{y=1-\sqrt{1-z}}^{y=1}\int_{z=0}^{z=1-y} xy+z\,\mathrm{dx}\,\mathrm{dy}\,\mathrm{dz} +$$

$$\int_0^1\int_0^{1-\sqrt{1-z}}\int_0^{\sqrt{1-z}} xy+z\ \mathrm{dx}\,\mathrm{dy}\,\mathrm{dz}$$

Triple Integrals in Cylindrical Coordinates

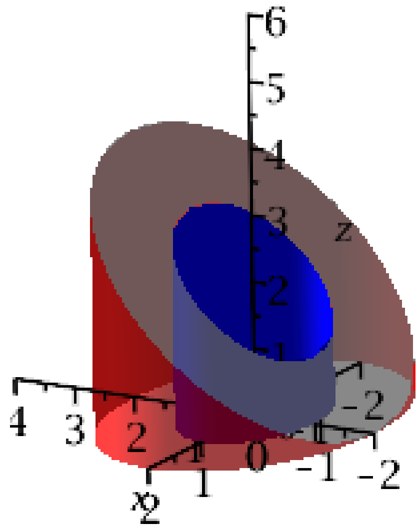
For double integrals, we sometimes needed to convert the problem to polar coordinates in order to solve it. A similar situations occurs with triple integrals. Instead of polar coordinates, we convert to what are called *cylindrical coordinates*. This was one of your word list words for today.

The key to determining whether a triple integral should be computed in cylindrical coordinates is to ask yourself the following:
If we fix *z* and project onto the *xy*-axis does the two dimentional region we get look like a polar coordinates problem?

An Example

Let E be the solid that lies between the cylinders $x^2+y^2=1$ and $x^2+y^2=4$, above the xy-plane, and below the plane $z=x+2$. We want to compute $\iiint_E y\,\mathrm{dV}$.

```
display(plot3d([x, sqrt(1-x^2), z], x=-2..4, z=0..x+2, color=blue, style=surface), plot3d([x, -sqrt(1-x^2), z], x=-2..4, z=0..x+2, color=blue, style=surface), plot3d([x, sqrt(4-x^2), z], x=-2..2, z=0..x+2, color=red, style=surface, transparency=0.25), plot3d([x, -sqrt(4-x^2), z], x=-2..2, z=0..x+2, color=red, transparency=0.25, style=surface), plot3d([x, y, x+2], x=-1..1, y=sqrt(1-x^2)..sqrt(4-x^2), color=gray, style=surface, transparency=0.25), plot3d([x, y, x+2], x=-1..1, y=-sqrt(4-x^2)..-sqrt(1-x^2), color=gray, style=surface, transparency=0.25), plot3d([x, y, x+2], x=1..2, y=0..sqrt(4-x^2), color=gray, style=surface, transparency=0.25), plot3d([x, y, x+2], x=-1..-2, y=-sqrt(4-x^2)..0, color=gray, style=surface, transparency=0.25), plot3d([x, y, x+2], x=-1..-2, y=sqrt(4-x^2)..0, color=gray, style=surface, transparency=0.25), plot3d([x, y, x+2], x=1..2, y=-sqrt(4-x^2)..0, color=gray, style=surface, transparency=0.25), axes=normal, labels=['x','y','z'])
```



We **need to add an extra *r* at the end, just like we did for polar coordinates.**

$$\int_0^{2\pi}\int_0^{2}\int_0^{r\cos\theta+2} r^2 \cos\theta \sin\theta \cdot r \, dz \, dr \, d\theta$$

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