Final Exam Review

This is just a guide to help you study. I do not guarantee that anything will or will not be on the exam based on this guide.

Basics

Wednesday, December 15 from 9-12 for the 8:30 AM section
Thursday, December 16 from 2-5 PM for the 2:30 PM section

No books, notes, cell phones. I will provide you with a scientific calculator and the Unit Circle Chart.

The exam will consist of two parts of roughly the same length (and with roughly the same distribution of difficulty of problems). The first part will skew toward the earlier material in the class and the second part will skew toward the later material in the class. You cannot leave the classroom until you have turned in the first part (unless it is an emergency!) and once you turn the first part in, you may go take a break until you are ready for the second part. Once you start the second part you again cannot leave the room until you finish it (barring an emergency again!). The two parts and the break cannot exceed 3 hours total.

Office hours

I will be in Zoom the following times. Please stop by to ask any questions!

- Monday, December 13 from 9:30 AM -10:30 AM
- Tuesday, December 14 from 9:30 AM- 12:00 PM
- Wednesday, December 15 from 1:15 PM - 3 PM
- Thursday, December 16 from 10 AM - 11:30 AM

Suggestions

- Work lots and lots of problems, especially those on material you don't understand as well. Try to solve problems without looking at the book for formulas or similar problems.
- When possible, ask yourself WHY you are solving a problem a certain way or WHY the result is true.
- Do not look at solutions unless you are desperate. It is much easier to read a correct solution than it is to figure it out yourself.
- Pay attention to details and check your work!!

Material

Functions

We dealt with polynomials, rational, trig, logarithmic, exponential, and absolute value functions, including:

- Graphs of basic functions
- How to find inverses and what inverses mean
- Continuity and differentiability
- Asymptotes
- The basics about trig functions and identities we use a lot (like $\sin^2 x + \cos^2 x = 1$)
- Inverse trig functions

Limits

Limits tell us about the behavior of a function as we get very close to a particular point (or infinity).

- Understand the statement of the $\epsilon - \delta$ definition of a limit.
- If you can use Direct Substitution for finite limits, do it!
- If there is some algebra you can do to simplify the function (such as canceling a term in the numerator and denominator), do that!
- If your function is a rational function, use the Limit Laws.
- If your limit satisfies one of the types where we can apply L'Hôpital's Rule, use it!
- Use $\lim_{x \to \infty} \frac{1}{x} = 0$ (Super Rule!) and $\lim_{x \to 0} \frac{1}{x} = \infty$ to analyze similar functions.

Derivative Rules

- Using the limit definition to find a derivative
- Power Rule (for polynomials)
- Product/Quotient Rule
- Trig and inverse trig functions
- Logs and exponential functions
- Chain Rule
- Implicit Differentiation

Applications of the Derivative

- Determine information about a function from the graph of its derivative
- How to find an equation for the tangent line
- Related Rates
- Optimization Problems
- Increasing-Decreasing Test and Concavity Test, First and Second Derivative Test

Integrals

- Why Riemann Sums represent area
- Fundamental Theorem of Calculus, parts I and II
- Definite Integrals
- Indefinite Integrals (antiderivatives) of polynomial, exponential, trig functions

- u-Substitution
- Integration by Parts
- Integration using Inverse Trig Functions

Applications of Integrals

- Area Between Curves
- Volumes
  - by revolution with disks
  - by revolution with washers
  - by cross sections other than circles
  - by cylindrical shells
- Average Value of a Function

Not on the Final

- Mean Value Theorem
- Squeeze Theorem
- Other bases besides e for log and exponential functions
- Using the $\epsilon - \delta$ definition to find limits directly (but see the "Limits" section above)
- Directly computing a Riemann Sum to find an integral (but see "Integrals" section above and be prepared to set up a problem using Riemann Sums)

Practice Problems

See the previous exam reviews for practice problems from the material covered on the first 3 exams.

- pg. 378 **Concept Check:** 1-5, 6(a)
- pg. 378-379 **Exercises:** 1-16, 21, 23-25, 26(a), 30
- pg. 482 **Concept Check:** 7, 8
- pg. 483-485 **Exercises:** 71-78, 89-90, 101-102, 105, 111-113, 116
- pg. 554 **Concept Check:** 1
- pg. 554-556 **Exercises:** 1-4, 7, 9, 10, 12, 17, 30

Sample Problems

These are questions from the material covered since the last exam. For sample problems for the material covered before that, see the previous exam review sheets.

(1) Find the integrals.

(a) $\displaystyle\int x e^{5x+2}\,\mathrm{d}x$

(b) $\displaystyle\int_1^4 \frac{e^{2x} - e^{4x}}{e^x}\,\mathrm{d}x$

(c) $\displaystyle\int \frac{\ln(\cos^{-1} x)}{(\cos^{-1} x)\sqrt{1 - x^2}}\,\mathrm{d}x$

(d) $\displaystyle\int x^2 \sin(x)\,\mathrm{d}x$

(2) (a) Compute $\int_{-3}^{3} \frac{x^2 \sin x}{x^4 + 1}\, dx$. **Hint**: What is $f(-x)$?

(b) What goes wrong if we try to compute $\int_{-3}^{3} \frac{x^2 \sin x}{x^4 - 1}\, dx$?

(3) Find the absolute maximum and minimum of the function $f(x) = x^2 - 8\ln x$ on the interval [1,5].

(4) Find the equation of the tangent line to the graph $f(x) = 2e^{1-x^2}$ at the point (1,2).

(5) Find the volume of the solid obtained by rotating the graph of $f(x) = e^x + 4$ from $x = 0$ to $x = 2$ about the x-axis.

(6) Find the volume of the solid obtained by rotating the region bounded by the graphs of $y = \frac{1}{x}, x = 1, x = 3$, and the x-axis about the line $x = -2$.

(7) Find the volume of the solid obtained by rotating the region bounded by the graphs of the functions $x = y^2$ and $x = 3y$ about the y-axis.

(8) A solid has as its base the region in the xy-plane bounded by the graphs of $x^2 = 16y$ and $y = 2$. Find the volume of the solid if every cross section by a plane perpendicular to the x-axis is a square.

(9) Find the area of the region bounded by the graphs of $y = \sqrt{x}$ and $y = x$ from $x = 0$ to $x = 4$.

(10) Find the average value of the function $f(x) = \dfrac{4(x^2+1)}{x^2}$ over the interval [1,3]. Explain where the formula for average value comes from.

(11) Find the limits.

(a) $\displaystyle\lim_{x\to\infty} \frac{x^{\frac{3}{2}} + 5x - 4}{x \ln x}$

(b) $\displaystyle\lim_{x\to\infty} x(e^{\frac{2}{x}} - 1)$