Exam 2 Review

This is just a guide to help you study. I do not guarantee that anything will or will not be on the exam based on this guide.

Basics

Wednesday, October 26, 2021 in class. No books or notes or cell phones. You may use a scientific calculator. Sections covered 3.5 - 3.8, 4.1-4.4, 4.5, and 4.7.

Practice Problems

- pg. 196 **Concept Check**: 8(g)-10
- pg. 197-199 **Exercises**: 13-43, 47-50 (not the normal line part of 49 and 50), 51ab, 53-54, 59-74, 76-80
- pg. 281 **Concept Check**: 1, 2a, 3-7
- pg. 282-284 **Exercises**: 1-11, 13-28, 36, 38, 40, 45, 46, 47, 48bc, 65ab

Suggestions

- Work lots and lots of problems, especially those on material you don't understand as well. Try to solve problems without looking at the book for formulas or similar problems.
- When possible, ask yourself WHY you are solving a problem a certain way or WHY the result is true.
- Do not look at solutions unless you are desperate. It is much easier to read a correct solution than it is to figure it out yourself.
- Pay attention to details and check your work!!

Sample Problems

(1) Differentiate the following functions.

(a) $f(x) = \tan(x + \tan x)$

(b) $g(r) = \sqrt{9 + r + \sin 3r}$

(c) $x(t) = (\cot(t^2))^5$

(2) Find $f'(x)$ by implicit differentiation for $4xy - \tan(y) = 3x^2 + \sin(x)$.

(3) Verify that the function $f(x) = 4 + \sqrt{x-1}$ on the interval [1,5] satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

(4) Use the following table of values to calculate the derivative of the given functions at $x = 2$.

x	$g(x)$	$h(x)$	$g'(x)$	$h'(x)$
2	5	4	-3	9
4	3	2	2	3

(a) $f(x) = \dfrac{g(x)}{h(x)}$
(b) $f(x) = g(h(x))$

(5) What is the equation for the tangent line of $x^2 + (3y)^2 = 13$ at the point (2,1)?

(6) A girl starts at a point A and runs east at a rate of 10 feet/second. One minute later, another girl starts at A and runs north at a rate of 8 feet/second. At what rate is the distance between them changing 1 minute after the second girl starts?

(7) Let $R(x)$ be a function that measures a company's revenue R from car sales (in thousands of dollars) in terms of advertising expenditures, x (also in thousands of dollars). Suppose the company is spending \$100,000 on advertising right now. If $R'(100) = -10$ should the company spend more or less on advertising to increase revenue? Why?

(8) A cone-shaped coffee filter of radius 6 cm and depth 10 cm contains water, which drips out through a hole at the bottom so the volume of the water in the filter decreases at a rate of 1.5 cm^3/sec. How fast is the water level falling when the depth is 8 cm? (Hint: the volume of a cone is $\dfrac{1}{3}\pi r^2 h$.)

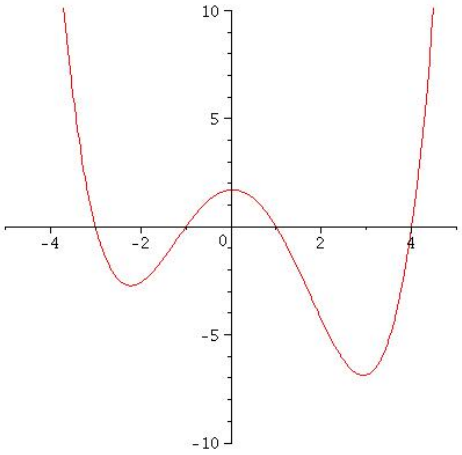
(9) Let $f(x) = \cos x + \frac{\sqrt{3}}{2}x$ on the interval $0 \leq x \leq 2\pi$.

(a) Find the critical number(s) of the function.
(b) Find the intervals on which f is increasing or decreasing.
(c) Use the first derivative test to find local maximum and minimum values of f.

(10) Let $f(x) = -x^4 + 2x^3 + x^2 - 2$.

(a) Find the critical number(s) of the function.
(b) Using the second derivative test, find the local maximum and minimum values of f.
(c) Find the interval(s) where the function is concave upward and concave downward.
(d) Find the inflection point(s).

(11) The graph of the **derivative** f' of a function f is shown below.



(a) On what intervals is f increasing or decreasing?
(b) At what values of x does f have a local maximum or minimum?

(12) (a) Find the x and y intercepts and any asymptotes of the function $f(x) = \frac{x^3 + 3x^2 - x - 3}{x^2 + 1}$

(b) Is $f(x) = x^4 - \cos(x)$ even, odd, or neither?

(13) Find the following limits.

(a) $\lim_{x\to\infty} \dfrac{3x^2 - x + 4}{5x - 7x^2}$

(b) $\lim_{x\to\infty} \dfrac{x^2}{x^3 - 3x^2 + x - 1}$

(c) $\lim_{x\to-\infty} \dfrac{4x^3 + 2x^2 - 1}{2x^2 - 1}$

(d) $\lim_{x\to\infty} \sqrt{5x^4 - 3x + 1} - \sqrt{5x^4 - 2x^2 + x + 4}$

(14) A cylindrical container, open at the top and of capacity 24π cubic inches is to be manufactured. If the cost of the material used for the bottom of the container is 6 cents per square inch, and the cost of the material used for the curved part is 2 cents per square inch, find the dimension which will minimize the cost. (Hint: The bottom of the cylinder is a circle and the curved part is really a rectangle--visualize cutting open a can and unfolding the curved part -- with height h and length the circumference of the bottom.)

(15) A poster of area 6000cm^2 has blank margins of width 10 cm on both the top and bottom and 6cm on each side. Find the dimensions that maximize the printed area.