Math 218: Combinatorics

Homework 8 : Due October 4

1. Prove that $n^2 \geq 3n$ for all $n \geq 3$.

2. Morris 6.2.6 #6.

3. Let x and y be in the set $\{\texttt{true}, \texttt{false}\}$ and let $x \oplus y$ denote the exclusive-or of x and y which is defined to be `true` if and only if exactly one of x and y is `true`. Note that the exclusive-or operation is associative, that is $a \oplus (b \oplus c) = (a \oplus b) \oplus c$. Prove by induction on n that $x_1 \oplus x_2 \oplus \cdots \oplus x_n$ is `true` if and only if an odd number of $x_1, x_2, \ldots, x_n$ are true.

4. Suppose M_i is an $r_{i-1} \times r_i$ matrix for r_i a positive integer and $1 \leq i \leq n$. So M_1 has r_0 rows and r_1 columns while M_2 has r_1 rows and r_2 columns. Prove that for all positive integers n, the matrix product $M_1 \cdot M_2 \cdots M_n$ is an $r_0 \times r_n$ matrix.

5. Bogart #73. Give a proof by induction of the binomial theorem. (Hint: Earlier in the semester, we learned some interesting relationships among the binomial coefficients.)