Math 218: Combinatorics

Homework 3 : Due September 10

1. A roller coaster has five cars, each containing four seats, two in front and two in back. There are 20 people ready for a ride. In how many ways can the ride begin? What if two of the people want to sit in different cars from each other?

2. Suppose a local restaurant offers a deluxe sandwich. There are three options for the bread, and you can pick up to two of the four meats, up to two of the three cheeses, and up to three of eight additional topics (tomatoes, peppers, etc.). How many different possible sandwiches are there? Assume one sandwich possibility is a piece of bread and no meat, no cheese, and no toppings.

3. Suppose that you are creating a password using 26 letters, 10 numbers, and 15 special characters. Each password must have exactly 6 letters, 2 numbers, and 2 special characters.
(a) How many passwords can you make if each character must be distinct?
(b) What if we allow repeated characters?

4. A bakery sells 6 different kinds of pastry. If the bakery has at least a dozen of each kind, how many different options are there for a box of a dozen pastries?

5. A classroom has two rows of eight seats each. There are 14 students, 5 of whom always sit in the front row, and 4 of whom always sit in the back row. In how many ways can the students be seated?

6. (a) (Bogart # 55) For $n \geq 1$, prove

$$\binom{n}{0} - \binom{n}{1} + \cdots + (-1)^n \binom{n}{n} = 0.$$

(b) Prove that for $n \geq 1$,

$$\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots = \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \cdots = 2^{n-1}.$$

7. By integrating the equation in the binomial theorem, prove that, for a positive integer n,

$$1 + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \cdots + \frac{1}{n+1}\binom{n}{n} = \frac{2^{n+1} - 1}{n+1}.$$