Math 218: Combinatorics

Homework 1 : Due September 1

Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on PWeb on the given day by 12:45 PM, or 7 PM if you LaTeX the assignment.

In Linear Algebra you likely learned the formal definition of an *injective (one-to-one)* and a *surjective (onto)* function. The first two questions ask you to use the basic proof techniques you learned in Linear Algebra for proving *for all*, *there exist*, and *if... then...* statements.

Suppose A and B are sets.

Definition. A function $f : A \to B$ is said to be *injective (one-to-one)* if whenever $f(a_1) = f(a_2)$ then $a_1 = a_2$.

You may find it useful to write the *contrapositive* of the injective definition. Sometimes that is easier to prove.

Definition. A function $f : A \to B$ is said to be *surjective (onto)* if for all $b \in B$, there is an $a \in A$ so that $f(a) = b$.

1. Suppose X, Y, and Z are sets and $f : X \to Y$ and $g : Y \to Z$ are functions.

   (a) Prove that if $g \circ f$ is injective then f is injective ($\circ$ just means function composition here).

   (b) Prove that if $g \circ f$ is onto then g is onto.

   (c) Find an example of functions $f : X \to Y$ and $g : Y \to Z$ so that $g \circ f$ is onto but f is not onto.

2. (a) Define f to be a function from pairs of natural numbers to the natural numbers defined as $f(m, n) = m^2 + n$. Prove that f is onto but not one-to-one.

   (b) Define g to be a function from pairs of integers to pairs of integers defined by $g(m, n) = (m+n, m-n)$. Prove g is not onto (might be a good idea to negate the definition of surjective) but is injective.

3. Problem #5 in Bogart. Make sure you write the solution up formally using Product or Sum rule explicitly when appropriate.