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# Math 218: Combinatorics

HOMEWORK LAST!!!!: DUE DECEMBER 10 \*

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1. Prove that any tree of order  $n$  contains a subtree (a subgraph which is also a tree) of order  $m$  for every  $m \leq n$ .
2. Let  $G$  be connected and let  $e$  be an edge of  $G$ . Prove that  $e$  is a bridge if and only if it is in every spanning tree of  $G$ .
3. Let  $e$  be an edge of  $K_n$ . Use Cayley's Theorem to prove that the graph with vertex set  $[n]$  and edge set that of  $K_n$  minus  $e$  has  $(n-2)n^{n-3}$  spanning trees.
4. Count the number of trees with vertex set  $[11]$  where all of the following hold (and, of course, explain your answer!):
  - (a)  $\deg(5) = 4$
  - (b)  $\deg(1) = \deg(7) = 3$
  - (c)  $\deg(4) = \deg(8) = 2$
  - (d) all the other vertices are leaves.

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\*Really due by 8 AM December 11