Math 7770 Spring 2011
Homework 1
Due: January 24, 2011

You are welcome to work together but everyone needs to write up **distinct** solutions. If you use any books outside of our textbook or other people, please make sure to give them credit. Make sure your solutions are complete. If your handwriting is atrocious, I am happy to give you a basic introduction to LaTeX.

Warmup

- §1.2 #2. Find the greatest common divisor g of the numbers 1819 and 3587 using the Euclidean Algorithm, and then find integers x and y to satisfy $1819x + 3587y = g$. (For the second part, check out Example 2 on pg. 13).
- §1.2 #5. How many integers between 100 and 1000 are divisible by 7?
- §1.2 #6. Prove that the product of three consecutive integers is divisible by 6; of four consecutive integers by 24.
- §1.2 #9. Show that if $ac \mid bc$ then $a \mid b$.
- §1.2 #10. Given $a \mid b$ and $c \mid d$, prove that $ac \mid bd$.
- §1.2 #17. Evaluate $(n, n+1)$ and $[n, n+1]$ where n is a positive integer.

Problems

1. §1.2 #14. Prove that if n is odd, $n^2 - 1$ is divisible by 8.
2. §1.2 #23. Prove that the square of any integer is of the form $3k$ or $3k+1$ but not of the form $3k+2$.
3. §1.2 #31. Let $n \geq 2$ and k be any positive integers. Prove that $(n-1) \mid (n^k - 1)$.
4. §1.2 #33. Prove that $(a, b) = (a, b, a+b)$, and more generally that $(a, b) = (a, b, ax + by)$ for all integers x and y.
5. §1.2 #43. Prove that $a \mid bc$ if and only if $\frac{a}{(a,b)} \mid c$.

Challenge

I. §1.2 #46. Prove that there are no positive integers $a, b, n > 1$ such that $(a^n - b^n) \mid (a^n + b^n)$.