MATH 232 Discrete Math

Homework 9

Basic Information

This assignment is due in Gradescope by 10 PM on the dates below.

Make sure you understand MHC honor code and have carefully read and understood the additional information on the class syllabus and the grading rubric. I am happy to discuss any questions or concerns you have!

You are always welcome to ask me for small hints or suggestions on problems.

Problems

Reading Problem 9M (Due: Sunday, April 12)

If you toss a coin five times in a row, what is the probability of getting two *Tails* and three *Heads*?

Wednesday Problems HW9 (Due: Wednesday, April 15)

Be sure you completely justify your answer using properties or results from class. An answer without justification will earn 0 points.

1. Suppose that you are creating a password using 26 letters, 10 numbers, and 15 special characters. Each password must have exactly 6 letters, 2 numbers, and 2 special characters.
   (a) How many passwords can you make if each character must be distinct?
   (b) What if we allow repeated characters?

2. A classroom has two rows of eight seats each. There are 14 students, 5 of whom always sit in the front row, and 4 of whom always sit in the back row. In how many ways can the students be seated?

3. In the following problems, assume $n \geq 1$.
   (a) Prove $\binom{n}{0} - \binom{n}{1} + \cdots + (-1)^n \binom{n}{n} = 0$. (The +1 and -1 terms alternate.).
   This is the result we needed in class in the proof of Inclusion-Exclusion.

(b) Prove $\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots = \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \cdots = 2^{n-1}$. Notice there are two different equal signs to prove here. (Hint: this is probably part (b) for a reason!)

4. Use *combinatorial reasoning* to prove that for n and k positive integers the following identity holds:

$$k\binom{n}{k} = n\binom{n-1}{k-1}.$$

5. Use c*ombinatorial reasoning* to prove the following identity for $1 \leq k \leq n$

$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}.$$

6. Suppose you toss a 6-sided die 10 times and record the number on the top of the dice each time. Use Inclusion-Exclusion to determine the number of ways those dice could be thrown so that each of the 6 numbers occur at least once in your list of 10 numbers. Here we assume tossing a 1 and then nine 6's is different than tossing nine 6's first and then a 1.

7. Determine the number of permutations of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ in which at least one odd integer is fixed.

Reading Problem 9F

No reading since there is an exam.