

# MATH 232 Discrete Math

## Homework 8

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### Basic Information

This assignment is due in Gradescope by 10 PM on the dates below.

Make sure you understand MHC [honor code](#) and have carefully read and understood the additional information on the [class syllabus](#) and the [grading rubric](#). I am happy to discuss any questions or concerns you have!

You are always welcome to ask me for small hints or suggestions on problems.

### Problems

#### Reading Problem 8M (Due: Sunday, April 5)

If a class has 11 students that say they like chocolate ice cream, 13 students that say they like vanilla ice cream, and 5 of the already mentioned students like both chocolate and vanilla, how many students are in the class?

#### Wednesday Problems HWS (Due: Wednesday, April 8)

Deck of card background in case you need it. A standard deck of cards has 52 cards. There are four suits (hearts ♥, spades ♠, diamonds ♦, and clubs ♣) and each suit has 13 cards (the numbers 2 through 10 which I call the “number cards”, the Ace, and the “face cards”: king, queen, and jack). A “hand” is the set of cards you are dealt in a game like poker.

**Be sure you completely justify your answer using properties or results from class. An answer without justification will earn 0 points.**

1. A sandwich shop in your hometown offers 3 different breads (white, wheat, gluten free), 4 fillings (ham, turkey, salami, and veggie mix), and 5 cheeses (feta, provolone, cheddar, American, and gouda). How many different sandwiches could you order? (Assume you must include bread, 1 filling, and 1 cheese).
2. In poker, a hand consists of five cards dealt to a player. “Four of a kind” means your hand consists of four cards of the same value (plus one other card). So for example, four 7s and a Jack or 4 Queens and a 8. How many different “four of a kind” hands are there where we do not care what order the cards are dealt to you?

3. How many ways are there to pick three cards out of a deck of 52 cards if our first choice must be a 6, the second choice must be a face card, and the third choice must be a “number card” from the hearts suit? Assume we do not return the already picked cards to the deck for the next choice, and assume the order we pick these in matters, i.e., we must pick a 6 first, and then a face card, and then a hearts number.
4. How many 3-letter “words” (strings of characters, they don’t actually have to be identifiable words) can you form from the letters of the word STRONG? How many of those words contain an “S”? (You may not use a letter more than once.)
5. How many integers greater than 5400 have **both** of the following properties:  
*(a) the digits are distinct, and (b) the digits 2 and 7 do not occur?*
6. A roller coaster has five cars, each containing four seats, two in front and two in back. There are 20 people ready for a ride.  
 (a) In how many ways can the ride begin?  
 (b) What if two of the people want to sit in different cars from each other?
7. Suppose the sandwich shop from the first problem offers a deluxe sandwich. There are three options for the bread, and you can pick up to two of the four meats, up to two of the three cheeses, and up to three of eight additional topics (tomatoes, peppers, etc.). How many different possible sandwiches are there? Assume one sandwich possibility is a piece of bread and no meat, no cheese, and no toppings.

**Reading Problem 8F** (Due: Friday, April 10)

Try to come up with a combinatorial proof to explain why the following equation is true:

$$n + (n - 1) + (n - 2) + \cdots + 3 + 2 + 1 = \binom{n + 1}{2}.$$

Hint: the right side counts the ways to pick out 2 elements from a set of  $n + 1$  elements. Can you figure out why the left side also counts that value in total? The left side has a bunch of “+” in it which means repeatedly applying the addition principle somehow.