MATH 232 Discrete Math
Homework 7

Basic Information

This assignment is due in Gradescope by 10 PM on the dates below.

Make sure you understand MHC honor code and have carefully read and understood the additional information on the class syllabus and the grading rubric. I am happy to discuss any questions or concerns you have!

You are always welcome to ask me for small hints or suggestions on problems.

Problems

Reading Problem 7M (Due: Sunday, March 29)

An ice cream shop has 12 flavors of ice cream. Say you want a cone with 3 scoops of ice cream. How many options are there? (Notice that bottom chocolate is different than top chocolate.) What if we stipulate that the flavors all need to be different?

Wednesday Problems HW7 (Due: Wednesday, April 1)

Be sure for the proof problems that you use the techniques and proof-writing guidelines we have talked about in class.

1. Suppose you want to mail a letter but all you have are five and six cent stamps. Determine (with proof) all possible postage values you can create with those five and six cent stamps. (For example, you could have an 11 cent postage by putting a 5 cent stamp and a 6 cent stamp on the letter.)

2. Let x and y be in the set {true, false} and let $x \oplus y$ denote the exclusive-or of x and y which is defined to be true if and only if exactly one of x and y is true. Note that the exclusive-or operation is associative, that is $a \oplus (b \oplus c) = (a \oplus b) \oplus c$.

Prove by induction on n that $x_1 \oplus x_2 \oplus \cdots \oplus x_n$ is true if and only if an odd number of $x_1, x_2, \ldots, x_n$ are true. (Notice this is induction plus an if and only if.)

4. What is wrong with the following proof where we claim (falsely) that $6n = 0$ for all integers $n \geq 0$.

   We prove this by induction. First, for the base case, let $n = 0$ then $6 \cdot 0 = 0$. Now suppose that $n > 0$. Write n as the sum of two integers both less than n and *greater than* 1*, so* $n = a + b$ *where* $1 < a, b < n$*. By the induction hypothesis* $6a = 0$ *and* $6b = 0$ *so* $6n = 6(a + b) = 6a + 6b = 0 + 0 = 0$*.*

5. A chocolate bar consists of n squares arranged in a rectangular pattern. You split the bar into small squares, always breaking along the lines between the squares. Use induction to prove that no matter how you break the bar, it takes $n - 1$ breaks to split it into the n smaller squares.

Comments: Chocolate bars are not necessarily one long line of rectangles. When $n = 6$ the bar could consist of 6 small squares in a row, or it could consist of two rows of 3 squares each. Here is a picture of a chocolate bar, and some physics on why they typically break at the seams.

6. Prove that if $n + 1$ integers are chosen from the set $\{1,\ 2,\ \ldots,\ 2n\}$ then there are always two which differ by 1.

7. Suppose a bag contains 100 apples, 100 strawberries, 100 peaches, and 100 kiwis. How many pieces of fruit must we pick out until we guarantee we have chosen at least a dozen pieces of fruit of the same kind? **Be sure to use results from class to justify this.**

Reading Problem 7F (Due: Thursday, April 2)

Why are the outer edges of Pascal's triangle all 1s? Why is the next number in (from the left and from the right) always n?