MATH 232 Discrete Math
Homework 9

Basic Information
This assignment is due in Gradescope by 10 PM on the dates below.

Make sure you understand MHC honor code and have carefully read and understood the additional information on the class syllabus and the grading rubric. I am happy to discuss any questions or concerns you have!

You are always welcome to ask me for small hints or suggestions on problems.

Problems

Reading Problem 9M (Due: Sunday, November 9)

If a class has 11 students that say they like chocolate ice cream, 13 students that say they like vanilla ice cream, and 5 that like both chocolate and vanilla, how many students are in the class?

Wednesday Problems HW9 (Due: Wednesday, November 12)

Be sure you completely justify your answer using properties or results from class. An answer without justification will earn 0 points.

1. A roller coaster has five cars, each containing four seats, two in front and two in back. There are 20 people ready for a ride.
   (a) In how many ways can the ride begin?
   (b) What if two of the people want to sit in different cars from each other?

2. Suppose a local restaurant offers a deluxe sandwich. There are three options for the bread, and you can pick up to two of the four meats, up to two of the three cheeses, and up to three of eight additional topics (tomatoes, peppers, etc.). How many different possible sandwiches are there? Assume one sandwich possibility is a piece of bread and no meat, no cheese, and no toppings.

3. Suppose that you are creating a password using 26 letters, 10 numbers, and 15 special characters. Each password must have exactly 6 letters, 2 numbers, and 2 special characters.
   (a) How many passwords can you make if each character must be distinct?
   (b) What if we allow repeated characters?

4. A classroom has two rows of eight seats each. There are 14 students, 5 of whom always sit in the front row, and 4 of whom always sit in the back row. In how many ways can the students be seated?

5. In the following problems, assume $n \geq 1$.

   (a) Prove $\binom{n}{0} - \binom{n}{1} + \cdots + (-1)^n \binom{n}{n} = 0$. (The +1 and -1 terms alternate.)

   (b) Prove $\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots = \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \cdots = 2^{n-1}$. Notice there are two different equal signs to prove here. (Hint: this is probably part (b) for a reason!)

6. Use *combinatorial reasoning* to prove that for n and k positive integers the following identity holds:
$$k\binom{n}{k} = n\binom{n-1}{k-1}.$$

7. Use *combinatorial reasoning* to prove the following identity for $1 \leq k \leq n$
$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}.$$

Reading Problem 9F (Due: Thursday, November 13)

Problem coming later!