Basic Information

This assignment is due in Gradescope by 10 PM on the dates below.

Make sure you understand MHC honor code and have carefully read and understood the additional information on the class syllabus and the grading rubric. I am happy to discuss any questions or concerns you have!

You are always welcome to ask me for small hints or suggestions on problems.

Problems

Reading Problem 7M (Due: Sunday, October 26)

If you have 5 shirts and 4 pairs of pants, how many different possible outfits do you have?

Wednesday Problems HW7 (Due: Wednesday, October 29)

Be sure for the proof problems that you use the techniques and proof-writing guidelines we have talked about in class.

1. Define $n!$ to be the *factorial function*, the product of all positive numbers up to and including n, so $n! = 1 \cdot 2 \cdots (n-1) \cdot n$. Prove by induction on $n \geq 4$ that $n! > 2^n$.

2. Determine (with proof) all possible postage we can create with five and six cent stamps.

3. Let x and y be in the set {true, false} and let $x \oplus y$ denote the exclusive-or of x and y which is defined to be true if and only if exactly one of x and y is true. Note that the exclusive-or operation is associative, that is $a \oplus (b \oplus c) = (a \oplus b) \oplus c$.

Prove by induction on n that $x_1 \oplus x_2 \oplus \cdots \oplus x_n$ is true if and only if an odd number of $x_1, x_2, \ldots, x_n$ are true. (Notice this is induction plus an if and only if.)

4. What is wrong with the following proof where we claim (falsely) that $6n = 0$ for all integers $n \geq 0$.

   We prove this by induction. First, for the base case, let $n = 0$ then $6 \cdot 0 = 0$. Now suppose that $n > 0$. Write n as the sum of two integers both less than n and *greater than* 1*, so* $n = a + b$ *where* $1 < a, b < n$*. By the induction hypothesis* $6a = 0$ *and* $6b = 0$ *so* $6n = 6(a + b) = 6a + 6b = 0 + 0 = 0$.

5. A chocolate bar consists of n squares arranged in a rectangular pattern. You split the bar into small squares, always breaking along the lines between the squares. Use induction to prove that no matter how you break the bar, it takes $n - 1$ breaks to split it into the n smaller squares.

Comments: Chocolate bars are not necessarily one long line of rectangles. When $n = 6$ the bar could consist of 6 small squares in a row, or it could consist of two rows of 3 squares each. Here is a picture of a chocolate bar, and some physics on why they typically break at the seams.

6. Suppose A is a set of 10 natural numbers between 1 and 100 (inclusive). Show that two different subset of A have the same sum.

(Hint: How many subsets of A are there? Since there are only 10 elements of A, and each of them is at most 100, how many different possible sums are there?)

7. Prove that if $n + 1$ integers are chosen from the set $\{1,\ 2,\ \ldots,\ 2n\}$ then there are always two which differ by 1.

Reading Problem 6F (Due: Thursday, October 30)

If we have a set with 6 elements in it, how many subsets have 1 element in them? How many have 5 elements in them?