Basic Information

This assignment is due in Gradescope by 10 PM on the dates below.

Make sure you understand MHC honor code and have carefully read and understood the additional information on the class syllabus and the grading rubric. I am happy to discuss any questions or concerns you have!

You are always welcome to ask me for small hints or suggestions on problems.

Problems

Reading Problem 5M (Due: Sunday, October 5)

In class we showed that for each $n \in \mathbb{Z}$ we have n^2 is either a multiple of 3 or a multiple of 3 plus 1. Restate this statement in terms of congruences instead.

Wednesday Problems HW5 (Due: Wednesday, October 8)

Be sure to use the techniques and proof-writing guidelines we have talked about in class.

1. (Old problem originally posted on HW 4) For this problem, you can simply write down each algebraic step you do. It is ok to NOT use any words in the solution to this problem! But make sure you show every step of the computation. (This is problem 5d in MR, pg 424.)
   (a) For the integers $a = 21361$ and $b = 12628$, use the Euclidean algorithm to find $\gcd(a, b)$.
   (b) Use the algorithm "backwards" to write the $\gcd(a, b)$ as a linear combination of a and b.

2. Let a, b, c, d, f be positive integers. If gcd(a, b) = d and gcd(a, c) = f and gcd(b, c) = 1, prove that gcd (d, f) = 1.

3. Let a, b be positive integers. Prove that if $\gcd(a, b) > 1$ then $b \mid a$ or b is not prime. (Hint: how do we prove *or* statements?)

4. In class, we defined a function $f : A \to B$ to be surjective if

for all $b \in B$ there exists an $a \in A$ so that $f(a) = b$

(a) Carefully write the negation of this statement.
(b) Use (a) to prove that the function $g : \mathbb{Z} \to \mathbb{Z}$ satisfying $g(n) = 2n + 1$ is not surjective.

5. We use the symbol $\mathbb{R}^+$ to mean all positive real numbers.
(a) Prove that the function $f : \mathbb{R}^+ \to \mathbb{R}^+$ defined as $f(x) = \sqrt{x}$ is injective.
(b) Prove that the same function in (a) is surjective.

6. Above you showed that $g : \mathbb{Z} \to \mathbb{Z}$ satisfying $g(n) = 2n + 1$ is not surjective. Instead prove that $h : \mathbb{R} \to \mathbb{R}$ satisfying $h(x) = 2x + 1$ is surjective.

Reading Problem 5F

No question this week! We have an exam instead.