MATH 232 Discrete Math

Homework 4 (updated Sept 26)

Basic Information

This assignment is due in Gradescope by 10 PM on the dates below.

Make sure you understand MHC honor code and have carefully read and understood the additional information on the class syllabus and the grading rubric. I am happy to discuss any questions or concerns you have!

You are always welcome to ask me for small hints or suggestions on problems.

Problems

Reading Problem 4M ~~(Due: Sunday, September 28)~~

We will talk about this problem Wednesday October 1 instead.

Define the function $d(n) : \mathbb{Z}^+ \to \mathbb{Z}^+$ to output the number of positive divisors of n, for any positive integer n. So for example, $t(2) = 2$ since 1 and 2 are both divisors of 2. Similarly $t(24) = 8$ since the divisors of 24 are the set $\{1, 2, 3, 4, 6, 8, 12, 24\}$ which has cardinality 8.

(a) Calculate $d(k)$ for each positive integer k from 1 through 12.

(b) Does there exist a positive integer n so that $d(n) = 1$? If so, say which n. If not, say why not.

(c) Does there exist a natural number n such math $d(n) = 2$? If so, say which n. If not, say why not.

Wednesday Problems HW4 (Due: Wednesday, October 1)

Be sure to use the techniques and proof-writing guidelines we have talked about in class.

1. Let $A = \{3x + 1 : x \in \mathbb{R}\}$ and $B = \{3x - 2 : x \in \mathbb{R}\}$. Prove that $A = B$.

2. Given two sets A and B, we say they are *disjoint* if they have no elements in common. We want to prove that, given any sets A and B, the sets $A \cap B$ and $A - B$ are disjoint.
   (a) How would we write down "for all", "there exist", and/or "if...then..." statements to precisely capture what it would mean for these two sets to be disjoint?
   (b) Now prove the statement(s) you wrote down in (a).

3. (a) Prove that if $ab \mid c$ then $a \mid c$ and $b \mid c$.

(b) For all $m, n \in \mathbb{Z}$ prove that the set $A = \{x \in \mathbb{Z} : m \cdot n \mid x\}$ is a subset of the set $B = \{x \in \mathbb{Z} : m \mid x\} \cap \{x \in \mathbb{Z} : n \mid x\}$.

4. We defined M_d to be the set of integer multiples of an integer d.
   (a) Prove that $M_{12} \subseteq M_6$.
   (b) Assume $a \mid b$. Prove that $M_b \subseteq M_a$.

5. The following claim is false:
   Let A, B, and C be subsets of some set U. If $A \nsubseteq B$ and $B \nsubseteq C$, then $A \nsubseteq C$.

(a) Here is a wrong "proof". Describe precisely where the logic fails in this proof.
   We assume that A, B, and C are subsets of U and that $A \nsubseteq B$ and $B \nsubseteq C$. This means that there exists an element $x \in A$ that is not in B and there exists an element x that is in B and not in C. Therefore, $x \in A$ and $x \notin C$, and we have proved that $A \nsubseteq C$.

(b) Suppose we are in a special case of (a) where the set U is the integers $\mathbb{Z}$. Come up with an explicit example for sets A, B, and C where the claim stated above fails. If you work with others to come up with ideas for this part, you should each have different final answers. Briefly say why your example is correct.

6. Prove that if n is an odd integer, then $n^2 = 8x + 1$ for some $x \in \mathbb{Z}$.

Reading Problem 4F (Due: Thursday, October 2)

(Old question from last week)
Find an example of functions f and g so that $g \circ f$ is surjective, and g is surjective, but f is not surjective. Your example can be ANY functions f and g. (This is one of the example mentioned on page 333 of MR.)