Math 510 Fall 2009
Homework 11 Extra Problem

Prove the following lemma which we used to prove that a matching M is a max matching of a graph G if and only if there is no M-alternating path in the graph.

Lemma 1. *Let M_1 and M_2 be matchings of the graph $G = (V, E)$ and let $E' = M_1 \cup M_2 - (M_1 \cap M_2) = (M_1 - M_2) \cup (M_2 - M_1)$. Then each connected component of $G' = (V, E')$ is one of the following three types:*

1. *a single vertex*
2. *a cycle with an even number of edges whose edges are alternately in M_1 and M_2*
3. *a path whose edges are alternately in M_1 and M_2 and whose two end vertices are each matched by one of M_1 or M_2 but not both.*

Hint: A connected component could be of type (1). So assume we have a connected component which is not a single vertex and try to prove it is of type (2) or type (3). To do this, consider bounds on the sum of the degrees in the connected component.