Research Project Ideas

Below are 7 ideas for project topics for this class. In each case, I only giving you some general ideas of a topics. There are many different directions you could choose to go with these projects. As you explore these topics, here are some good general sources of information:

- Miranda has “Further Reading” sections at the end of every chapter.
- The bibliographies of any of the books we used have many sources.
- MathSciNet (we will talk about this resource in class on Feb 27).
- Searching the Grinnell library catalog for “Riemann surfaces” gives a lot of different books about Riemann surfaces.

Action: By 1 PM Tuesday, March 5 (really 8 AM March 6) fill out the Qualtrics form linked on PWeb.

I. Complex Tori

We’ve been introduced to complex tori already but there is a lot more to say about them. Miranda keeps returning to them as examples throughout the book. There are many different avenues this topic could explore including content in Miranda we won’t cover.

II. Hyperelliptic Riemann Surfaces

Hyperelliptic Riemann surfaces are special families of Riemann surfaces introduced in Miranda on page 60. They are one way of generalizing complex tori. Cavalieri & Miles also talk about them in a slightly different way, introduced in section 6.3 (that section assumes a bit more topology than we’ve learned yet). As with complex tori, there are numerous different avenues this topic could lead to.

III. Curves in Positive Characteristic.

We will learn soon that compact Riemann surfaces are precisely smooth projective curves in the complex plane. Also, you may recall from abstract algebra that the character of a ring is the order of the 1 element, if it is a finite number. (For $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{C}$ the order is infinite, and we say that ring is *characteristic 0*.) There is a lot to say about curves over rings that are not characteristic zero. Appendix A in Cavalieri and Miles is a good place to start.

IV. Algorithms Riemann Surfaces

There are specialized computer packages that can be used to compute different things related to Riemann surfaces. One project could be to deeply explore several of the algorithms. Students would have to do more than just explain a couple algorithms, I would expect some coding, demonstrations of how the code works, comparison of different algorithms, etc. Part II (Ch 2 and 3) of the book "Computational approach to Riemann Surfaces" which you can find online through our library is a good place to start.

V. Riemann Existence Theorem and Monodromy

We haven't learned about groups acting on Riemann surfaces yet, but basically a finite group G can act on a Riemann surface by "moving" the points of the surface around and this leads to an equivalence relation on the points on X. Points that go to each other under the action of the group can be considered as one equivalence class and this set of equivalence classes produces a quotient space X/G which is also a Riemann surface. There are some very interesting results, such as Riemann Existence Theorem which tell us when a particular group can act on a particular compact Riemann surface. Section 6.2 of Cavalieri and Miles talks about some of these results, and Chapter III.3 and III.4 in Miranda (some of which we will cover after spring break) talk about these ideas as well.

VI. Calculus on Riemann Surfaces.

In class, will not talk extensively about doing calculus on Riemann surfaces (I instead chose to focus the latter content in the class on group actions on Riemann surfaces.) But there is a lot of calculus-like ideas (like integration) that translate well to Riemann surfaces. Some sources to consider are Chapter IV of Miranda (he talks about Stokes Theorem, Residue Theorem, etc). There are also harmonic functions which show up in several places in Miranda which I have skipped, and Donaldson's "Riemann Surface" book (available online through our library) focuses more on the analytic side of Riemann surfaces.

VII. Applications of Riemann Surfaces to Physics.

This is an area I know very little about so students working in this area would need to have a very clear plan of what they wanted to study. The content must be first and foremost about the mathematics and secondarily about the physics. A couple of possible starting points are Appendix D in the Cavlieri and Miles book, and the introduction in "An Introduction to Riemann Surfaces, Algebraic Curves and Moduli Spaces" by Schlichenmaier (available online through our library).