Math 321: Foundations of Abstract Algebra

Homework 9 : Due April 17

1. 18.5
2. 18.15
3. 18.23 Note that (b) asks you to prove $\phi(I)$ is maximal if and only if $\phi(I)$ is proper ideal of S and *then* asks you to prove that if ker $(\phi) \subseteq I$ then $\phi(I)$ is maximal.
4. An ideal A of a commutative ring R with unity is said to be finitely generated if there exist elements $a_1, a_2, \ldots a_n$ of A such that $A = (a_1, a_2, \ldots, a_n)$. An integral domain R is said to satisfy the ascending chain condition if every strictly increasing chain of ideals $I_1 \subset I_2 \cdots$ must be finite in length. Show that an integral domain R satisfies the ascending chain condition if and only if every ideal of R is finitely generated.
5. 19.10
6. 19.12 We used this in the proof of Theorem 19.3.
7. 19.13 We used this in the proof of mod p irreducibility.
8. (a) How many roots does $X^2 + \overline{3}X + \overline{2}$ have in $\mathbb{Z}/6\mathbb{Z}$?

   (b) Find, with proof, all the irreducible polynomials of degree 2 or 3 in $\mathbb{Z}/2\mathbb{Z}[X]$.

   (c) Show that the polynomial $\overline{2}X + \overline{1}$ in $\mathbb{Z}/4\mathbb{Z}[X]$ has a multiplicative inverse in $\mathbb{Z}/4\mathbb{Z}[X]$.
9. (a) If I is an ideal of a ring R, prove that $I[X]$ is an ideal of $R[X]$.

   (b) Let R be a commutative ring with unity. If I is a prime ideal of R, prove that $I[X]$ is a prime ideal of $R[X]$.