Math 321: Foundations of Abstract Algebra

Homework 8 : Due April 10

1. 16.15

2. 16.23

3. 17.2

4. 17.18

5. 17.20

6. 17.22

7. 17.25

8. Let $S = \{a + bi \mid a, b \in \mathbb{Z}, b \text{ even}\}$. Prove that S is a subring of $\mathbb{Z}[i]$ but not an ideal of $\mathbb{Z}[i]$. See exercise 16.24 for the definition of $\mathbb{Z}[i]$.

9. (a) Give an example of a ring that has exactly two distinct maximal ideals.

   (b) Suppose that R is a commutative ring and $|R| = 30$. If I is an ideal of R and $|I| = 10$, prove that I is maximal ideal.

   (Hint: Rings are abelian groups.)