Math 321: Foundations of Abstract Algebra

Homework 7 : Due April 3

1. 12.19 (This completes a piece of a proof from class.)
2. Define $\phi : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ by sending (x, y) to $x + y$.

   (a) Prove that ϕ is a surjective homomorphism.

   (b) What is the kernel of this mapping? Be sure to use words.
3. 13.5
4. Let $G = \left\{ \begin{bmatrix} a & 2b \\ b & a \end{bmatrix} \mid a, b \in \mathbb{Q}. \right\}$ and let $H = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$

   (a) Prove that G and H are isomorphic under addition.

   (b) Notice that G and H are closed under multiplication (you can just check this for yourself). Does your isomorphism preserve multiplication as well as addition?
5. 13.8
6. If g and a are elements of a group G, prove that $Z(a)$ is isomorphic to $Z(gag^{-1})$ (where $Z(a) = \{h \in G \mid hah^{-1} = a\}$ is the *centralizer* of the element a and not to be confused with the *center* of a group, see page 93).
7. 16.1 (Make sure you prove it from the most basic definitions.)
8. 16.9 (b) and (c) (Part (a) should look familiar!)
9. 16.11

Extra

1. 16.24 (The mathematical concepts in this problem form some of the core concepts of a field of math called Algebraic Number Theory.)