Math 321: Foundations of Abstract Algebra

Homework 2 : Due February 7

1. Suppose a and b are integers that divide the integer c. If a and b are relatively prime, prove that ab divides c. Show, by example, that if a and b are not relatively prime, then ab need not divide c.

2. Prove that every prime greater than 3 can be written in the form $6n + 1$ or $6n + 5$.

3. # **4.2**. Also, prove that G is a group.

4. These will be 2 points each. Be sure to explain your work.
   (a) # **4.5**
   (b) # **4.7**

5. # **4.12** (Remember, even if a question in the book technically has a yes/no answer, you need to justify your answer with a mathmatical proof for credit.)

6. # **4.19** (Be sure to consider finite and infinite order elements.)

7. # **4.20**

8. Assume G is an **abelian group** for this problem.
   (a) Prove that $(xy)^n = x^n y^n$ for all x and $y \in G$.
   (b) # **4.32** (Hint: One approach is to prove the result if $(m, n) = 1$ first, and then find a way to use that result and the Fundamental Theorem of Arithmetic to prove the case when $(m, n) > 1$.)