Math 321: Foundations of Abstract Algebra

Homework 11 : Due May 8

This assiignment must be turned in by 8 AM Saturday, May 9.

1. 21.6
2. (a) Assume n is an even positive integer and show that D_n acts on the set consisting of pairs of opposite vertices of a regular n-gon.
   For example, if $n = 6$ label the vertices $\{a, b, c, d, e, f\}$ in order around the hexagon. Then the set A would be: $\{(a, d), (b, e), (c, f)\}$ and r would act on those vertices by $r * (a, d) = (b, e)$ or $r * (c, f) = (a, d)$.

   (b) Find the kernel of this action.
3. Let G be a group and let $G = A$.

   (a) Prove that if G is non-abelian then the map defined by $g*a = ag$ for all $g, a \in G$ does not satisfy the axioms of a group action of G on itself.

   (b) Prove that the map defined by $g * a = ag^{-1}$ does satisfy the axioms of a group action of G on itself.
4. Define A to be the set of ordered pairs with entries from the set $\{1, 2, 3\}$,
   $$A = \{(i, j) \mid 1 \leq i, j \leq 3\}.$$
   Let S_3 act on A by taking a $\sigma \in S_3$ and defining $\sigma * (i, j) = (\sigma(i), \sigma(j))$. So if $\sigma = (1\ 2)$ then $\sigma * (1, 3) = (2, 3)$ and if $\sigma = (1\ 2\ 3)$ then $\sigma * (1, 3) = (2, 1)$.

   (a) Find the orbits of S_3 on A.

   (b) For each orbit $\mathcal{O}$ from (a), pick some $a \in \mathcal{O}$ and find the stabilizer of a in S_3.
5. If the center of G is of index n, prove that every conjugacy class has at most n elements.
6. Find all conjugacy classes for the following groups. What is the Class Equation for each? Show your work, and say a few words about your process.

   (a) Q_8

   (b) D_5

   (c) $S_3 \times \mathbb{Z}/2\mathbb{Z}$

   Note: Theory is your friend here! Try to minimize the number of computations you must do.
7. Find (with proof) all finite groups which have exactly two conjugacy classes.