MAT 218 Elementary Number Theory
2022 Fall

Basic Information

Instructor: Prof. Jen Paulhus paulhus@math.grinnell.edu

> I do not check email between 9 PM and 7 AM on weekdays, and only infrequently on the weekends. I will make every effort to respond to emails within 24 hours.

My Office Hours: Mondays 11:00 - 11:30 AM, Tuesday 1:30-2:30 PM, Wednesdays 11:00 - 11:30 AM, Thursdays 2:00 -3:00 PM, *or by appointment*

Student Leader: Isidora Bailly-Hall baillyha@grinnell.edu

Isidora's Office Hours: See PWeb for a rotating schedule

Class Webpage: http://paulhus.math.grinnell.edu/teaching/ma218f22.html

Class Meetings: 10-10:50 AM Monday, Wednesday, Friday, Noyce 2517

Text: *Explorations in Number Theory* by Jeanne Agnew (available on PWeb)

What is Number Theory?

Number theory is one of the oldest branches of mathematics, far older than Calculus. The subject, at its most basic, asks questions about the integers. For instance, how are the prime numbers distributed among the integers? What are integer solutions to a particular polynomial equation? Which integers can be expressed as the sum of two squares? The tools used to solve these problems span topics in much of modern pure mathematics.

Learning Goals

The primary goal of this class is to prepare you for the 300-level courses and beyond. We will use combinatorics as a backdrop to discuss how to think about mathematics, how to read mathematical writing, and how to write good mathematics. **Perhaps for the first time in your math career the goal is to learn how to *produce* mathematics much more than how to *use* mathematics.**

Here is what a successful student will master in this class.

- Develop individualized study skills in preparation for the foundations courses by devising personalized strategies to:
  - get the most out of class time,
  - best utilize time spent on homework,
  - prepare for exams at this level, and
  - successfully internalize the material.
- Strengthen writing and communication skills in mathematics to the point where your writing clearly articulates the key ideas to non-experts.
- Hone problem-solving skills: learn how to approach problems in a new field and how to attack different types of problems.

- Learn key concepts of number theory: divisibility, congruences, Euler ϕ-function, quadratic residues, and multiplicative functions. Learn general mathematical concepts: induction, equivalence relations, and basic counting techniques.

Growth towards these goals will be measured in class by the student's ability to
- Solve new never before seen problems during exams,
- Solve increasingly difficult homework problems,
- Write clear solutions to homework problems which may be understood by peers, and
- Know definitions, basic computational techniques, and fundamental examples in the subject.

By the end of this course, you should have a much better sense of what advanced mathematics entails. Even if your life goals do not involve further mathematical study, many of the skills you will learn translate to other fields. As this course is a prerequisite for higher level computer science courses, we will also cover material in preparation for those courses.

Success in my Classes
Students come to this class was very different backgrounds, skills, and experiences. Usually the most successful students in my class have two things in common: they work hard, and they are able to self reflect honestly and then make adjustments accordingly.

My job is to help you *all* learn number theory and prepare for the 300-level math class. I do not think any less of you if you struggle with the material, or if you come ask me for help in office hours. In fact, I view struggling and discomfort with material as an essential part of learning! If you are frustrated or overwhelmed with the course, email me and we'll set up a time to talk.

Grading Policies

Homework
Problem sets are 25% of your grade. Assignments will be posted on the class webpage. Each homework assignment is worth the same percent of your final grade, although point totals will vary from assignment to assignment. Homeworks are **due on PWeb by 9:45 AM on their posted due date** (roughly every other class period). Any part of the assignment which is LaTeX'd may be turned in by 7 PM the same day (if you are in class that day). I will drop your lowest homework score. Early in the semester, some re-writes on homework solutions will be assigned, and for those early assignments, no late homework will be accepted. After the first exam, I will allow for up to 2 late homeworks per student. This means each student can turn in up to 2 assignments up to 48 hours late as long as they let me know they will be doing so before the initial due date of the particular assignment. See the grading rubric for more information and suggestions for mathematical writing, as well as tips for solving math problems at this level.

Exams
There will be three in-class exams scheduled for **Friday, September 23, Wednesday, October 26,** and **Monday, November 21**. No notes, no books, and no calculators. No make up exams will be given, unless agreed to beforehand so contact me immediately if you have any conflict with an exam. The highest exam scores will be 15% of your final grade, and the other the two exams will count as 10% of your final grade.

Final
There will be a cumulative final which will count for 18% of your grade. The final exam is **Friday, December 16 from 9 AM - 12 PM**.

Reading Reports
To get the most out of our class time, we want our brains to already be thinking about the material. As such, it will be a substantial benefit to you if you have done the assigned reading *before* each class. To demonstrate that you are keeping up with the readings, up to twice a week there will be a brief report for you to file about the reading material for the upcoming class. The report will take the form of a few questions in a Qualtrics form, **due by midnight the night before the class**. Links to the forms will be on the class webpage. There is a password for the forms, posted on PWeb. I will generally grade your responses for completion, not correctness, and cumulatively they will count for 15% of your final grade.

Engagement
The only way to learn mathematics is to engage deeply with it, so 7% of your grade will reflect how well you are engaging in the class. Your contributions to discussions in class, and your questions in Isidora's mentor session or my office hours impact this part of your grade. If you are regularly late for class, your engagement grade will suffer.

MASSS
There will be a number of opportunities to attend talks by students or visitors in the department. These talks give you a chance to hear new math and meet students and faculty in the department. If you attend at least 4 of the talks and email me a brief paragraph within 2 days of each talk, I will add 1/2 point to your final letter grade.

Class Mentor
The mentor for Math 218, Isidora Bailly-Hall, is a junior math major who has taken both of our 300-level foundations courses, and has participated in summer research projects away from Grinnell. Her job is to be a guide for students as they transition to proof-based mathematics courses, including discussing her own experiences transitioning to 300-level courses. Isidora will also be available to talk about proofs from class or the readings, as well as discuss alternative proofs to and provide supplemental information about material in class. She can help with homework and your writing, particularly by pointing out where a completed solution goes wrong, or where a written solution gets hard to follow. She also can provide some basic LaTeX support. Isidora will have a rotating schedule which will be posted on PWeb.

Class Policies

Homework Submission

I highly recommend you learn to LaTeX (if you haven't already). Otherwise you will need to scan your homework in **black and white into one .pdf file** and then submit it via PWeb. Do not take pictures of the homework and email those. Picture photos are very large and are hard for me to annotate.

Homework Returns

All homework assignments will be returned to you on OneDrive. I will create and share a folder with each of you.

Workload

The amount of time students spend on this course outside of class varies depending on many factors, but about 8-10 hours a week beyond the classroom time is quite typical.

Cellphones

Our class time together is limited and valuable. Cellphone usage should be restricted to only what is essential for this class. Don't be shoe shopping during class!

Accommodations

Grinnell College makes reasonable accommodations for students with documented disabilities. Students need to provide documentation to the Coordinator for Disability Resources, located on the ground level floor of Steiner Hall (641-269-3124) and discuss your needs with them. Students should then notify me within the first few days of classes so that we can discuss ways to ensure your full participation in the course and coordinate your accommodations.

Academic Honesty

Make sure you are familiar with the college's guidelines for academic honesty.

For this class, you are allowed (and even encouraged) to work together to solve homework problems but everyone must write their own solutions. Here are some more explicit instructions regarding this: if several students are sitting around discussing how to solve a problem and in the course of the conversation one of you figures out a key piece and discusses that piece with everyone else, then you may all go off on your own and write up your own answers. Additionally, you may discuss your written solutions with anyone you have worked with to solve a particular problem.

However, it is not ok if one of you solves the problem yourself first, and then tells other people the key pieces of the problem. If you have already figured out a solution to a problem, and someone asks you how to solve the problem, tell them you already figured the problem out and they should talk to me or Isidora. Giving good hints is sometimes very tough. Conversely, if you know your colleagues have figured out a problem, you should not ask them for help but instead should talk to me or Isidora

Consulting any completed solution is academically dishonest.

I repeat: **consulting any completed solution is academically dishonest.** Never search the Internet for a solution to a problem. Reading a math solution is much easier than figuring it out yourself. It only hurts your learning to find solutions online or in another book. I intentionally write some difficult problems and do not assume students will ace the problem sets. Talk to me or to Isidora or other students in the class who have not yet solve the problem if you're stuck.

As mentioned above, giving a hint that helps but doesn't give away too much information is very hard. As such you should not ask for homework help from students who have formerly taken the class (except the mentor!) or who are currently in other sections of MAT218.

There are very serious consequences if you are found to be in violation of one of these policies. A typical first offense is a zero on the particular assignment, your final grade in the course is dropped a full or part of a letter grade, and you are ineligible to receive honors from any department.

The Pandemic and its aftermath

Most of us thrive with structure and clear goals we can work towards. I will, therefore, set explicit deadlines for each homework, and enforce homework and exam deadlines. I anticipate this will be beneficial for almost all of you.

That being said, these are still strange and difficult times. If anything comes up which makes it hard for you to keep up with the work in this class (e.g., illness, family obligations, etc.), let me know and I can work with you to make another plan.

We are all going to have to deal with unexpected interruptions, technology not working right, and just generally bad days. **It's imperative that you keep up good communication with me if you run into issues or have questions.**

Unsolicited Advice
- Take ownership of your education.
- One major goal of the classes to teach you how to understand the basic language of mathematics and logic. Like any new subject, this requires much practice, practice, practice.
- Keep up with the reading assignments. Reread the material again later.
- Spend time thinking about how you learn, and evaluating what is working and not working for you in your academic life.
- Most of you will be challenged during the semester. Be prepared to not "get" everything right away.
- If you are struggling, make an appointment to talk with me.