Math 218: Elementary Number Theory

Homework 8 : Due October 5

1. Let r and s be real numbers. Define $r \sim s$ whenever $r - s$ is an integer.

   (a) Prove that this is an equivalence relation.
   (b) Describe what the equivalence classes look like (you should say more than just what the definition says, describe what the equivalence classes "look" like).

2.1 #9. Prove that if $M_m + \{r_1\} \neq M_m + \{r_2\}$ then they are disjoint (i.e. $M_m + \{r_1\} \cap M_m + \{r_2\}$ is empty). This problem shows that the residue classes form a *partition* of the integers.

2.2 #1. If $a \equiv b \bmod m$ and $c \equiv d \bmod m$, prove that $ax + cy \equiv bx + dy \bmod m$ for any integers x and y.

2.2 #2. Prove that $10^k \equiv 1 \bmod 9$ for every integer $k > 0$.

2.2 #9. In ordinary arithmetic, if $a^2 = b^2$, then $a = \pm b$. Is the analogous statement true in the ring of residues mod m, i.e. if $a^2 \equiv b^2 \bmod m$ does that mean $a \equiv \pm b \bmod m$? If it is true, prove it. If it is not true, explain a specific counterexample that fails.

2. Suppose that you are creating a password from 26 letters, 10 numbers, and 15 special characters. How many such 10-character passwords are possible if they must have exactly 6 letters, 2 numbers, and 2 special characters, and they must consist of 10 distinct symbols in the password (so *P1ssw0rd!!* is not a legitimate password because it contains two s's and two !'s)?