# Math 218: Elementary Number Theory

Homework 7 : Due September 30

**Note**: There will no longer be rewrite opportunities starting with this homework. Make sure you follow all the mathematical norms for writing we've learned about in the last month.

1.9 #7. If $f$ is multiplicative and $f(a) \neq 0$ for some $a \neq 0$, prove that $f(1) = 1$.

1. (a) [1.11 #1.] Do on your own but you do not need to submit this part (and feel free to practice more examples before answering part b). Calculate $\sigma(72)$, $\sigma(250)$ and $\sigma(8000)$.

   (b) [1.11 #7.] For what values of $n$ does $\sigma(n) = b$ if:

   (1) $b = 14$ (2) $b = 15$

   (3) $b = 16$ (4) $b = 18$

2. Determine whether each of the following relations is reflexive, symmetric, and transitive (you should check each individual property, not all three at once). If a certain property fails, you should give a specific counterexample. (This problem will be worth 6 points.)

   a. $S = \mathbb{Z}$ where $a \sim b$ means $a - b \neq 1$.

   b. $S = \mathbb{Z}$ where $a \sim b$ means that both $a$ and $b$ are even.

   c. $S = \mathbb{Z}$ where $a \sim b$ means $a \mid b$.

3. A friend tries to convince you that the reflexive property is redundant in the definition of an equivalence relation because they claim that symmetry and transitivity imply it. Here is the argument they propose:

   *"If $a \sim b$, then $b \sim a$ by symmetry, so $a \sim a$ by transitivity. This gives the reflexive property."*

   Now you know that their argument must be wrong because one of the examples in Problem 2 is symmetric and transitive but not reflexive. Pinpoint the error in your friend's argument. Be as explicit and descriptive as you can.