Math 218: Elementary Number Theory

Homework 5 : Due September 19

Note: *There will be no rewrite opportunities on this homework due to the upcoming exam.*

§1.7 #8. (a) Prove that the equation $ax + by = n$ has a solution for all integers n when $(a, b) = 1$.

(b) State a necessary condition for there to be an integer solution to the equation in (a) if $(a, b) = d \neq 1$. By "necessary condition" mathematicians mean finding a statement blah to fill in the sentence

When $(a, b) = d \neq 1$, if blah is true then there is an integer solution to the equation in (a).

Explain why your condition works. (*Suggestion: Try a few specific examples first to come up with a condition.*)

§1.7 #9. See book. Assume that the student needs to use all \$200 for each part of this problem. The second part is really saying at least 6 math books. (Remember, our textbook is from the early 1970s so prices in this problem reflect that, as do specific gender pronouns!)

§1.8 #4. If $(m, n) = 1$, prove that $(m + n, mn) = 1$.

§1.8 #11. If $(a, n) = d$ and $(r, n) = 1$, prove that $(r - a, d) = 1$.

§1.10 #1&2. You do not need to include words for this problem.

(a) Write in standard form these four numbers: 286, 390, 1278, 842

(b) Write the product represented by $\prod_{p|1260} p^{a_p}$.

§1.10 #7. A unitary divisor of a number n is a divisor d having the property that $(d, n/d) = 1$. Write the unitary divisors of $n = p^2q^5$, where p and q are primes. Explain your answer.

§1.10 #14. This is a optional bonus question if you are interested in the idea of unique factorization failing. It will be worth only a couple bonus points.