# Math 218: Elementary Number Theory

Homework 16 : Due December 2

7.2 #5. (a) Compute $(\mu * \phi)(12)$.

(b) Prove that for all number theoretic functions $f$ that

$$(\mu * f)(p^k) = f(p^k) - f(p^{k-1}).$$

7.3 #6. Let $f$ be the characteristic function of the set of odd integers and $g$ be the characteristic function of the set of even integers.

(a) Compute $(f * g)(16)$, $(f * g)(840)$, and $(f * g)(231)$.

(b) Determine (with proof) what $(f * g)(n)$ is for any positive integer $n$. Your answer will likely depend on the factorization of $n$.

7.4 #8. (a) Prove that $(\phi * \tau)(p^a) = \sigma(p^a)$ for any prime $p$.

(b) Use (a) and results from class to prove for all $n$ that $(\phi * \tau)(n) = \sigma(n)$. (This part of the problem is really just about putting pieces together. You should not have to prove anything from scratch in this part.)

1. (a) Prove that $\mu(d)/d$ is a multiplicative function.

   (b) Use (a) to prove that

$$\phi(n) = n \sum_{d|n} \frac{\mu(d)}{d}.$$

(Hint: We know that multiplicative functions are completely determined by their values on powers of primes.)