Math 218: Elementary Number Theory

Homework 15 : Due November 28

2.5 #4. In this problem we will reprove Theorem 2.5.1 in a different way than in the book or in class. Don't assume Theorem 2.5.1 anywhere in this problem.

(a) For an arbitrary prime power p^k, determine what the following sum is.

$$\sum_{d|p^k} \phi(d).$$

(b) Now define the function $F(n)$ as $F(n) = \sum_{d|n} \phi(d)$. We proved ϕ is multiplicative already and we know $F(n)$ is multiplicative by Theorem 1.11.3, so use that fact and the value you found in (a) to determine what $F(n)$ is.

2.5 #5. For p prime, prove that $\sigma(p) + \phi(p) = p\tau(p)$.

7.1 # 3. (a) Prove that the function $\omega(n)$ is additive but not completely additive. This function was defined in class (and in example 7.1.1) as the number of distinct primes that divide n.

(b) Is the function defined as $\nu(n) = a_1 + a_2 + \cdots + a_k$ where $n = p_1^{a_1} p_2^{a_2} \cdot p_k^{a_k}$ additive? Why or why not? If it is, is it completely additive?

7.1 # 6. Use induction and the definition of an additive function to prove Theorem 7.1.1 (Be sure to also prove the assertion that $f(1) = 0$.)