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# Math 218: Elementary Number Theory

HOMWORK 12 : DUE NOVEMBER 4

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1. Suppose you toss a 6-sided dice 10 times and record the number on the top of the dice each time. Use Inclusion-Exclusion to determine the number of ways those dice could be thrown so that each of the 6 numbers occur at least once in your list of 10 numbers. Here we assume tossing a 1 and then nine 6's is different than tossing nine 6's first and then a 1.

2. Use Inclusion-Exclusion to determine the number of permutations of the set  $\{1, 2, \dots, 9\}$  in which at least one odd integer is fixed.

A *permutation* is an arrangement of a list of  $n$  elements, so for  $n = 3$  the 6 permutations are:  
1, 2, 3    1, 3, 2    2, 1, 3    2, 3, 1    3, 1, 2    3, 2, 1.

Here we say 1, 2, 3 and 1, 3, 2 both fix 1, while 1, 2, 3 and 3, 2, 1 both fix 2.

3. Suppose  $n$  students go to the DHall for dinner and they leave their coats outside. How many ways can the students pick up their coats after dinner so that no one gets their original coat back? Your final answer will have  $n$  in it.

- 3.1 #2. Find the solution(s) of the following congruences by hand (don't just search for solutions, be clever!)

(a)  $5x \equiv 6 \pmod{21}$

(b)  $47x \equiv -15 \pmod{21}$

(c)  $7x \equiv 3 \pmod{50}$

(d)  $11x \equiv 21 \pmod{59}$