
Math 215: Linear Algebra

WRITING ASSIGNMENT 1 : DUE NOVEMBER 3

Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by 3 PM Grinnell Time, or 7 PM if you LaTeX the assignment.

This is a writing assignment. You should treat this like you would a writing assignment in an English or Philosophy or History course, in the sense that everything you write should be part of a complete sentence and part of larger paragraphs which each serve a clear purpose, your grammar and spelling should be accurate, and (if you handwrite it) you should not have crossed out sections where you change your mind about what you want say. For each problem you should plan out how you want to write it in an outline or draft, and then write a polished, final product to be submitted. Your audience should be fellow students who are excited about math but have not learned linear algebra yet. You are welcome to ask a friend who is not in the class to read your answer and let you know if it makes sense to them. There is no page or paragraph limits for this assignment but you should be (1) thorough and complete and (2) concise and exact.

This first assignment is primarily to give you practice writing careful proofs. The mathematics is not necessarily difficult (and some of the proofs might not be very long to write) but you should focus on writing very carefully. No symbols as proxies for perfectly good English words, make sure you write complete sentences (which always end with a period!), etc. I will be grading these problems and giving you careful feedback.

1. Pythagorean triples are 3 integers a, b, c such that $a^2 + b^2 = c^2$. You probably saw these numbers in a geometry course. For example 3, 4, 5, is a Pythagorean triple since $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.

For this question, you will answer the question, “Are there Pythagorean triples of the form $m, m + 7, m + 8$?” To answer this question you should:

- (a) write what you believe is the correct *for all* mathematical statement or *there exists* mathematical statement to represent the answer, and then
- (b) carefully prove the statement from (a) using the proof techniques for *for all* or *there exist* statements which we have been working on in class.

If you aren't sure where to start, I suggest playing around a bit with some Pythagorean triples. Can you create some yourself? Can you think of a general way to create them?

2. Prove or disprove the following two statements.
 - (a) There exists an $x \in \mathbb{Z}$ so that for all $y \in \mathbb{Z}$ we have $y^2 > x$.
 - (b) For all $x \in \mathbb{N}^+$ there exists a $y \in \mathbb{N}^+$ such that $xy = 1$.