Math 215: Linear Algebra

Problem Set 9 : Due November 20

(20 points) Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by 3 PM Grinnell Time, or 7 PM if you LaTeX the assignment.

1. (3 points) Show that if $[T] = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ then $T \circ T = R_\pi$ (where R_π is rotation by 180°).

2. (a) (3 points) Let $A = \begin{pmatrix} \frac{9}{10} & \frac{3}{10} \\ \frac{3}{10} & \frac{1}{10} \end{pmatrix}$. Compute $A \cdot A$ and simplify your answer as much as possible. **Show all your work**.

   (b) (3 points) Explain your answer to (a) by geometrically interpreting the matrix A as a certain linear transformation. (Suggestion: Can you recognize this matrix as one from the families of transformations we discussed?)

3. (5 points) One result that is "missing" from Proposition 2.6.6 is the idea that matrix multiplication is commutative. This is stated as:

   For all 2×2 *matrices* A *and* B*, we have* $AB = BA$.

   However, this is a false statement. Using our work from early in the semester, negate this statement and then write a proof of the negation. You should include all your matrix computations in your proof.

4. Another "missing" idea from the algebra of matrices is that of *cancellation*. If x, y, and z are in $\mathbb{R}$, and we know that $xy = xz$, then we can cancel the x and conclude that $y = z$. This is not true for matrices. Let

   $$A = \begin{pmatrix} 3 & -6 \\ -1 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} -1 & 1 \\ 3 & 4 \end{pmatrix}, \qquad C = \begin{pmatrix} -3 & -5 \\ 2 & 1 \end{pmatrix}.$$

   (a) (3 points) Compute AB and AC and conclude that the products are the same. Show all your work.

   (b) (3 points) Explain what (a) tells us about *cancellation* with matrices.

5. (DON'T TURN THIS ONE IN) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be any 2×2 matrix. Definition 2.6.3 in the textbook defines $r \cdot A$ as $\begin{pmatrix} ra & rb \\ rc & rd \end{pmatrix}$.

   Prove Proposition 2.6.4 which says that if $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $r \in \mathbb{R}$, then $[r \cdot T] = r \cdot [T]$.