Math 215: Linear Algebra

Problem Set 4 : Due November 6

(20 points) Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by 3 PM Grinnell Time, or 7 PM if you LaTeX the assignment.

1. In class, we defined a function $f : A \to B$ to be injective when *for all* $a_1, a_2 \in A$, *if* $f(a_1) = f(a_2)$ *then* $a_1 = a_2$.

   (a) (1 point) Write the contrapositive of this definition. (We will sometimes use this as an alternative way to describe an injective function).

   (b) (1 point) Negate the definition of injective.

   (c) (2 points) Use (b) to prove that $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = \sin(x)$ is not injective.

2. (5 points) For this problem do a **double containment proof** to show the two sets $A = \{3x + 1 : x \in \mathbb{Z}\}$ and $B = \{3x - 2 : x \in \mathbb{Z}\}$ are equal. Anything other than a double containment proof will get no credit.

3. (5 points) Suppose A and B subsets of some set U. Prove or disprove that the sets $A \cap B$ and $A \backslash B$ are disjoint.

4. (6 points) The following claim is false:

   Let A*,* B*, and* C *be subsets of some set* U*. If* $A \not\subseteq B$ *and* $B \not\subseteq C$*, then* $A \not\subseteq C$*.*

   (a) Here is a wrong "Proof". Describe precisely where the logic fails in this proof.

   We assume that A*,* B*, and* C *are subsets of* U *and that* $A \not\subseteq B$ *and* $B \not\subseteq C$*. This means that there exists an element* $x \in A$ *that is not in* B *and there exists an element* x *that is in* B *and not in* C*. Therefore,* $x \in A$ *and* $x \notin C$ *, and we have proved that* $A \not\subseteq C$*.*

   (b) Suppose U is the integers, $\mathbb{Z}$. Come up with an explicit example for sets A, B, and C where the claim stated above fails. If you work with others to come up with ideas for this part, you should each have different final answers here.