Math 215: Linear Algebra

Problem Set 2 : Due November 2

(2o points) Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by 3 PM Grinnell Time, or 7 PM if you LaTeX the assignment.

1. (4 points) Negate the following statements. You do not need to prove or disprove the statements, simply write the negation so that no "not" appears.
   (a) For all $x \in \mathbb{R}$, we have $x^3 \geq x^2$.
   (b) There exists an $x \in \mathbb{Z}$ such that for all $y \in \mathbb{Z}$ we have $2x + 3y = 1$.

2. (12 points) Prove or disprove each of the following.
   (a) For all x in $\mathbb{R}$ we have $x^2 + 1 = (x + 1)^2$.
   (b) There exists an integer n so that $n > 3$ and $n < -5$.
   (c) There exists an n in $\mathbb{N}$ such that $n^2 - 1 = 0$.
   (d) There exists an x in $\mathbb{R}$ such that $\sin x = \cos x + 3$.

3. (4 points) Below is the proof of the statement

   For all $a \in \mathbb{Z}$ we have $2a^5 + 6a^3 - 4a + 3$ is odd.

   I have deleted some pieces of the proof. Either fill in each of the blank spaces to make the proof correct or write your own proof from scratch (following the conventions we've learned in class and which are talked about in the book). **Please underline or color differently the filled in blanks on your submitted answer if you choose to just fill in blanks.**

   ____________________________________. Then $2a^5+6a^3-4a+3 =$ _________________________ Since _________________________ is an integer, then _________________________ by the definition of odd numbers. Since a was arbitrary, we have proven the result.