Math 215: Linear Algebra

Problem Set 10 : Due November 23

(24 points) Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by 3 PM Grinnell Time, or 7 PM if you LaTeX the assignment. **For this assignment, show all of your computational work.**

1. Let T be the linear transformation with $[T] = \begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$.

   (a) (3 points) Show that if $\vec{v}$ lies along the line in the direction of $\binom{1}{1}$ then $T(\vec{v}) = \vec{0}$.

   (b) (3 points) Show that the range of this function is $\mathrm{Span}(\binom{1}{2})$.

   (c) (2 points) Find a matrix $B \neq 0$ so that $B \cdot [T] = 0$.

   (d) (2 points) Find a matrix $C \neq 0$ so that $[T] \cdot C = 0$.

2. (4 points) Determine $\mathrm{Null}(P_{\vec{w}})$ where $\vec{w} = \binom{1}{1}$. (Recall $P_{\vec{w}}$ is projection onto the vector $\vec{w}$.) Suggestion: Double containment is helpful.

3. (6 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation with standard matrix $[T] = \begin{pmatrix} 12 & 8 \\ -3 & -2 \end{pmatrix}$. Find vectors $\vec{u}$ and $\vec{w}$ with $\mathrm{Null}(T) = \mathrm{Span}(\vec{u})$ and $\mathrm{range}(T) = \mathrm{Span}(\vec{w})$.

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation with standard matrix $[T] = \begin{pmatrix} 4 & 1 \\ -2 & -1 \end{pmatrix}$

   (a) (2 points) Why do we know T has an inverse? (Explicitly refer to any Theorems or Propositions you use.)

   (b) (2 points) Compute $T^{-1}\left(\binom{2}{3}\right)$.